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Technical note

On the derivation of a tensor to calculate six degree-of-freedom, musculotendon joint stiffness: Implications for stability and impedance analyses

Joshua G.A. Cashaback [a,*], Jim R. Potvin [a], Michael R. Pierrynowski [b]

[a] Department of Kinesiology, McMaster University, 1280 Main Street West, Hamilton, ON, Canada L8S 2K1
[b] School of Rehabilitation Science, McMaster University, Hamilton, ON, Canada L8S 2K1

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ABSTRACT

Major joints, such as the knee, shoulder, and spine, can buckle along the translational degrees-of-freedom (DoF), causing injury to ligaments and other passive tissues. Despite this, stability and impedance analyses have focused primarily on the rotational DoF. As such, mathematical models quantifying musculotendon translational stiffnesses remain limited and, to our knowledge, there are no published works that explicitly describes the interactions between DoF. Using an energy approach, we derived a six DoF stiffness tensor and provided the necessary equations needed to quantify the musculotendon stiffness of any joint. Using a knee model, we then compared the derived stiffness tensor against two commonly used measures: one that excludes translational DoF and another that excludes interactions between DoF. We found that both of these measures had large over-estimations of stiffness, particularly for the rotational DoF, compared to our derived tensor. These findings indicate that previous analyses may have found rotational DoF to be stable when they were unstable.

1. Introduction

Both joint stability (Bergmark, 1989; Crisco and Panjabi, 1991; Cholewicki and McGill, 1996; Potvin and Brown, 2005) and joint impedance (Hogan, 1984; Lee et al., 2011) analyses depend on the quantification of joint stiffness. Although, all human joints have six-degrees-of-freedom (DoF)—three rotational and three translational, almost all research implementing these analyses have focused solely on the rotational DoF and do not include the translational DoF. Without any loading, the knee, shoulder, and spine translate an average of 8.7 mm (Walker et al., 1988), 1.9 mm (Graichen et al., 2000), and 1.4 mm (Boden and Wiesel, 1990), respectively, during passive motion. With larger shearing forces, these joints can translate much further, potentially causing passive tissue damage or joint dislocation (Fleming et al., 1993; Lippitt et al., 2003; Howarth, 2011). Despite strong empirical evidence suggesting that muscles can provide joint stiffness and prevent translational motion (Hirokawa et al., 1991), there have been limited attempts to quantify muscular translational stiffness (Oosterom et al., 2003; Cashaback et al., 2013). Furthermore, while mathematical models have included interactions between rotational DoF (Crisco and Panjabi, 1991; Cholewicki and McGill, 1996), we are unaware of any work that explicitly defines the interactions between DoF. Given the importance of joint stability, further work is needed to rigorously define the musculotendon stiffness matrix (i.e., stiffness tensor) for all six DoF.

In this short communication, we derive the explicit equations for a tensor that can be used to quantify the musculotendon joint stiffness. Using the knee joint as an example, we will demonstrate the importance of including all six DoF and their interactions when quantifying musculotendon joint stiffness, by comparing the results to previous analysis methods.

2. Methods

By modeling an individual musculotendon unit as a spring, we can define its energy storage (Cholewicki and McGill, 1996) as

$$u_i = f_i \delta l_i + \tfrac{1}{2} k_i \delta l_i^2, \tag{1}$$

where k_i, f_i, u_i, and δl_i, represent some individual musculotendon's short-range stiffness (N/mm), force (N), stored elastic energy (J) and change in length, respectively, along its line-of-action (LoA). To determine δl_i, we must geometrically define the muscle length prior to (l_0) and following (l_1) a virtual displacement, which is an infinitesimal positional change with time being held constant. Fig. 1a and b, respectively, shows a pure translational and rotational displacement of a muscle coordinate (A; insertion) to a new position (A'), relative to muscle coordinate (B; proximal node).

For the example shown in Fig. 1a, the coordinate A' can be calculated as the original muscle coordinate $A(A_x, A_y, A_z)$ plus a translation (x) along the x-axis. This can be summarized in parametric form as $(A_x', A_y', A_z') = (A_x + x, A_y, A_z) = (A_x, A_y, A_z) + (x, 0, 0)$. For the pure rotational virtual displacement, it is sufficient to assume that the movement from A to A' is linear and tangential to the path of the circle arc (Fig. 1b). Furthermore, it is sufficient to assume that the virtual distances

* Corresponding author. Tel.: +1 905 525 9140x20175; fax: +1 905 525 6011.
E-mail address: cashabjg@mcmaster.ca (J.G.A. Cashaback).

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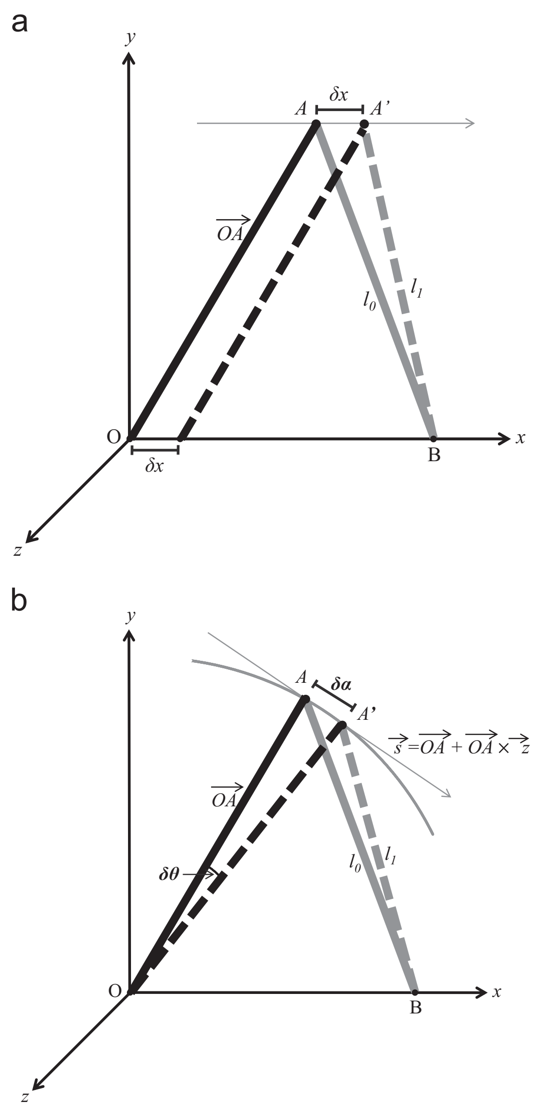

Fig. 1. Coordinate A is moved to a new position, A', following (a) an infinitesimal translational perturbation (δx) along the x-axis and (b) an infinitesimal rotational perturbation ($\delta\theta$), approximated with the tangential vector $\vec{s}$, about the z-axis. The origin, O, represents the instantaneous joint center-of-rotation. In (b), note the difference between the arc circle and tangent $\vec{s}$, from point A to A', is indistinguishable (i.e., $\delta\theta \cdot \overrightarrow{OA} \approx \delta\alpha$). The change in muscle length (δl) from the original (l_0) to the perturbed position (l_1) causes storage (or release) of elastic energy.

traveled along the tangent and circle arc are equal in magnitude (i.e., $\delta\theta \cdot \overrightarrow{OA} \approx \delta\alpha$). To define $\vec{s}$, we take the cross product of vectors $\overrightarrow{OA}$ and $\vec{z}$, where $\vec{z}(0,0,1)$ is a standard basis vector along the z-axis, and then add $\overrightarrow{OA}$. The cross product ($\overrightarrow{OA} \times \vec{z}$), whose ordering conforms to the 'right-hand-rule' convention, creates a vector that is perpendicular to both $\overrightarrow{OA}$ and $\vec{z}$. For Fig. 1b, we now define the virtual rotation from point A to A', in parametric form, as $(A_x{'}, A_y{'}, A_z{'}) = (A_x + A_y \cdot \alpha, A_y - A_x \cdot \alpha, A_z) = (A_x, A_y, A_z) + \alpha(\overrightarrow{OA} \times \vec{z})$. Here, α represents the displacement magnitude along vector $\vec{s}$. It is easy to combine both of these pure movements and extend these concepts to include additional, orthogonal DoF. The hyperplane equations, in compact (Eq. (2)) and expanded (Eqs. (3a), (3b), and (3c)) parametric form, that accounts for the 6DoF virtual displacement—three translational and three rotational—of any point, are

$$(A_x^{'}, A_y^{'}, A_z^{'}) = (A_x, A_y, A_z) + (x, y, z) + \gamma(\overrightarrow{OB} \times \vec{x}) + \beta(\overrightarrow{OB} \times \vec{y}) + \alpha(\overrightarrow{OB} \times \vec{z}) \tag{2}$$

and

$$A_x^{'} = A_x + x + 0 \cdot \gamma - A_z \cdot \beta + A_y \cdot \alpha \tag{3a}$$

$$A_y^{'} = A_y + y + A_z \cdot \gamma + 0 \cdot \beta - A_x \cdot \alpha \tag{3b}$$

$$A_z^{'} = A_z + z - A_y \cdot \gamma + A_x \cdot \beta + 0 \cdot \alpha, \tag{3c}$$

respectively. In Eqs. (2) and (3), x (anterior/posterior), y (superior/inferior), z (medial/lateral), and γ (valgus/varus), β (axial), α (flexion/extension) represent the movement, for small displacements, along and about the x-, y-, z-axes, respectively. This follows the international biomechanics society convention for larger, finite movements (Wu and Cavanagh, 1995). Now that we have explicitly defined the 6DoF virtual movement of any point, we can use Eq. (2) or Eq. (3) to determine a change in muscle length following a virtual displacement as

$$\begin{aligned} \delta l_i &= l_1 - l_0 \\ &= [(A_x^{'} - B_x)^2 + (A_y^{'} - B_y)^2 + (A_z^{'} - B_z)^2]^{1/2} \\ &- [(A_x - B_x)^2 + (A_y - B_y)^2 + (A_z - B_z)^2]^{1/2}. \end{aligned} \tag{4}$$

By inserting Eq. (4) into Eq. (1), we now have an equation that computes the instantaneous $u_i(f_i, k_i, x, y, z, \gamma, \beta, \alpha)$ in a muscle following a virtual perturbation along any of the 6DoF.

The first- and second-order partial derivatives of $u_i(f_i, k_i, x, y, z, \gamma, \beta, \alpha)$, with respect to generalized coordinates (x, y, z, γ, β, α), have important, physical properties. The first-order partial derivatives, Maclaurin series approximated, form

$$J(u)_i = \left[\frac{\partial u}{\partial \gamma}\ \frac{\partial u}{\partial \beta}\ \frac{\partial u}{\partial \alpha}\ \frac{\partial u}{\partial x}\ \frac{\partial u}{\partial y}\ \frac{\partial u}{\partial z}\right], \tag{5}$$

where $J(u)_i$ is the Jacobian matrix of some musculotendon. The first three terms of $J(u)_i$ are some musculotendon's moment (N m) about the x–y–z axes, while the last three terms represent its force (N) along these axes. Performing the second-order partial derivatives, Maclaurin series approximated, yields the following Hessian matrix:

$$H(u)_i = \begin{bmatrix} \frac{\partial^2 u}{\partial \gamma^2} & \frac{\partial^2 u}{\partial \gamma \partial \beta} & \frac{\partial^2 u}{\partial \gamma \partial \alpha} & \frac{\partial^2 u}{\partial \gamma \partial x} & \frac{\partial^2 u}{\partial \gamma \partial y} & \frac{\partial^2 u}{\partial \gamma \partial z} \\ \frac{\partial^2 u}{\partial \beta \partial \gamma} & \frac{\partial^2 u}{\partial \beta^2} & \frac{\partial^2 u}{\partial \beta \partial \alpha} & \frac{\partial^2 u}{\partial \beta \partial x} & \frac{\partial^2 u}{\partial \beta \partial y} & \frac{\partial^2 u}{\partial \beta \partial z} \\ \frac{\partial^2 u}{\partial \alpha \partial \gamma} & \frac{\partial^2 u}{\partial \alpha \partial \beta} & \frac{\partial^2 u}{\partial \alpha^2} & \frac{\partial^2 u}{\partial \alpha \partial x} & \frac{\partial^2 u}{\partial \alpha \partial y} & \frac{\partial^2 u}{\partial \alpha \partial z} \\ \frac{\partial^2 u}{\partial x \partial \gamma} & \frac{\partial^2 u}{\partial x \partial \beta} & \frac{\partial^2 u}{\partial x \partial \alpha} & \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 u}{\partial x \partial y} & \frac{\partial^2 u}{\partial x \partial z} \\ \frac{\partial^2 u}{\partial y \partial \gamma} & \frac{\partial^2 u}{\partial y \partial \beta} & \frac{\partial^2 u}{\partial y \partial \alpha} & \frac{\partial^2 u}{\partial y \partial x} & \frac{\partial^2 u}{\partial y^2} & \frac{\partial^2 u}{\partial y \partial z} \\ \frac{\partial^2 u}{\partial z \partial \gamma} & \frac{\partial^2 u}{\partial z \partial \beta} & \frac{\partial^2 u}{\partial z \partial \alpha} & \frac{\partial^2 u}{\partial z \partial x} & \frac{\partial^2 u}{\partial z \partial y} & \frac{\partial^2 u}{\partial z^2} \end{bmatrix}, \tag{6}$$

where $H(u)_i$ is the symmetric stiffness tensor of some musculotendon. All the equations for the first and second order partial derivatives, found in matrices $J(u)_i$ and $H(u)_i$, are presented in Appendix A.

The musculotendon moment and force of a joint is simply found by summating the individual $J(u)_i$, such that

$$\begin{aligned} J(U) &= \sum_{i=1}^{n} J(u)_i \\ &= [M_x\ M_y\ M_z\ F_x\ F_y\ F_z], \end{aligned} \tag{7}$$

where $J(U)$ contains the musculotendon moments ($M_{x,y,z}$) and forces ($F_{x,y,z}$) of a joint, i is some musculotendon, and n is the total number of musculotendon units. Similarly, we find the musculotendon joint stiffness by

$$\begin{aligned} H(U) &= \sum_{i=1}^{n} H(u)_i \\ &= K, \end{aligned} \tag{8}$$

where $H(U)$ is the musculotendon joint stiffness tensor (K). The principal stiffnesses (PS) of tensor K can be found through singular value decomposition, such that

$$K = U\Sigma V^*, \tag{9}$$

where both U and V^* are unitary matrices and Σ is a diagonal matrix that contains the PS. Since K is a square, symmetric matrix, the singular values of Σ and the columns of U are equivalent to the eigenvalues and eigenvectors of K, respectively. Singular value decomposition, however, is more numerically stable than eigenvalue decomposition (Soderkvist and Wedin, 1993).

We obtained lower leg musculotendon coordinates and architecture from OpenSim (Musculographics Inc.; Arnold et al., 2010). The model was statically positioned in one of the two upright postures: (1) with the knee flexed 0° (ankle and hip flexed 0°), and (2) with the knee flexed 30° (ankle and hip flexed 15°). In each posture we took the A (tibial insertion) and B (proximal node) coordinates of the thirteen musculotendon units that crossed the knee, and transformed them into a tibial reference frame (Wu and Cavanagh, 1995).

To find each musculotendon's force (f_i) and stiffness (k_i) along its LoA, we used the distribution-moment approximation (DMA) model (Ma and Zahalak, 1991), incorporated with a nonlinear tendon compliance function and an active muscle force–length relationship from Thelen (2003). Briefly, the DMA-model solves four, coupled differential equations to calculate a muscle's instantaneous length, stiffness, force, and energy. To demonstrate the derived equations, we theoretically set the neural input of each musculotendon to maximum (r=1) (Cashaback et al., 2013; Brown and Potvin, 2007). For more information on the DMA-model, refer to Appendix B.

After k_i, f_i, A, and B were defined for each musculotendon, we calculated all the second-order partial derivatives found in Eq. (8) to calculate the musculotendon stiffness of the knee (tibiofemoral) joint. We then compared the PS of the full