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A finite viscoelastic–plastic model for describing the uniaxial ratchetting of soft biological tissues

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ABSTRACT

In this paper, a phenomenological constitutive model is constructed to describe the uniaxial ratchetting (i.e., the cyclic accumulation of inelastic deformation) of soft biological tissues in the framework of finite viscoelastic-plasticity. The model is derived from a polyconvex elastic free energy function and addresses the anisotropy of cyclic deformation of the tissues by means of structural tensors. Ratchetting is considered by the evolution of internal variables, and its time-dependence is described by introducing a pseudo-potential function. Accordingly, all the evolution equations are formulated from the dissipation inequality. In numerical examples, the uniaxial monotonic stress–strain responses and ratchetting of some soft biological tissues, such as porcine skin, coronary artery layers and human knee ligaments and tendons, are predicted by the proposed model in the range of finite deformation. It is seen that the predicted monotonic stress–strain responses and uniaxial ratchetting obtained at various loading rates and in various loading directions are in good agreement with the corresponding experimental results.

1. Introduction

The biomechanical behavior of soft biological tissues, i.e., skin, tendon, ligament, brain, cartilage and blood vessel, is necessary to be realized because it is playing a crucial role in the mechanical integrity analysis of the living body. Such a topic has attracted much attention in the last few decades and many great achievements have been made by numerous researches including the in vivo or in vitro tests and constitutive modeling. As reviewed by Fung (1993), soft biological tissues are typical highly functional composite materials mainly made of collagen and elastin proteins which bring some specific mechanical performances, such as non-linear anisotropic biomechanical response (generally non-linear transversely isotropic) and its time-dependence. The experimental studies on the anisotropic properties of soft biological tissues were performed by monotonic uniaxial and biaxial tests, as done by von Maltzahn et al. (1984), Yin et al. (1987), Silver et al. (2003), Holzapfel et al. (2005), Alastrué et al. (2008), Muñoz et al. (2008), Lokshin and Lanir (2009), Ma et al. (2010), Sommer et al. (2010), and many others. Also, the time-dependent responses of soft biological tissues were further investigated by the monotonic tensile tests at various strain rates or creep tests (Arumugam et al., 1994; Pioletti et al., 1996; Zeng et al., 2001; Shergold et al., 2006; Kettaneh et al., 2007). Accordingly, plenty of phenomenological constitutive models were developed to simulate the experimental phenomena mentioned above. The anisotropy was described reasonably by introducing structural tensors into the constitutive models (e.g., Humphrey and Yin, 1987a, 1987b; Ehret and Itskov, 2007; Peña and Doblaré, 2009; Peña et al., 2010; E. Peña et al., 2011; J.A. Peña et al., 2011; Maher et al., 2012); and the viscosity can be modeled by adopting a viscous potential function (Germain, 1973), the formulations of which were further developed by Pioletti et al. (1998), Pioletti and Rakotomanana (2000), and more recently by Roan and Vemaganti (2011).

A cyclic test within 3–30 cycles was often considered as a preconditioning, whose effect on the sequent biomechanical responses of the soft biological tissues was extensively discussed by Fung (1993), and the cyclic preconditioning resulted in repeatable data in the subsequent biomechanical tests. However, the cyclic biomechanical responses are also very important in assessing the fatigue life and wear property of soft biological tissues which should be investigated in detail. Some researchers were devoted to investigate the cyclic deformation of soft biological tissues experimentally and theoretically, as done by Emery et al. (1997), Carew et al. (2000), Sverdlik and Lanir (2002), Rubin and Bodner (2002), Nava et al. (2004), Giles et al. (2007), Kang and Wu (2011), Ehret and Itskov (2009) and Maher et al. (2012). Among these researches, more attentions were paid to the cyclic stress–strain responses of the tissues obtained under the stretch- or

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strain-controlled cyclic tests. Recently, the cyclic deformations of soft biological tissues were also investigated under the force- or stress-controlled cyclic loading conditions, as done by Kang and Wu (2011) and Achilli et al. (2012). Kang and Wu (2011) performed a detailed experimental observations to the cyclic deformation of porcine skin and named the cyclic accumulation of inelastic deformation occurred under the force-controlled cyclic loading conditions as the ratchetting of porcine skin. According to Kang and Wu (2011), the ratchetting of porcine skin is strongly dependent on the loading directions, levels and rates. The ratchetting of porcine skin and its time-dependence have not been addressed in the existing constitutive models of soft biological tissues.

Therefore, a new phenomenological viscoelastic–plastic constitutive model is constructed to describe the uniaxial ratchetting of soft biological tissues in this work. The proposed model is fully anisotropic, three-dimensional and thermodynamically consistent. The model is derived from a poly-convex elastic free energy function and addresses the anisotropy by means of structural tensors. The ratchetting is described by the adopted evolution equations of internal variables, and its time-dependence is described by introducing a pseudo-potential function. Finally, the capability of the proposed model to predict the monotonic stress–strain responses and uniaxial ratchetting of some biological tissues is verified by comparing the predictions with corresponding experimental results obtained from the literature (i.e., Holzapfel et al., 2005; Pioletti et al., 1996; Kang and Wu, 2011).

2. Constitutive model

2.1. Kinematics

Considering a homogeneous deformable body $\mathbf{B}$, with $\mathbf{X}$ an arbitrary position vector of a material particle in a fixed reference configuration and $\mathbf{x}$ the corresponding space position vector denoted by $\mathbf{x}=\chi(\mathbf{X},t)$ at current time t, the corresponding deformation gradient tensor, velocity vector and velocity gradient tensor are written, respectively, as

$$\mathbf{F}=\frac{\partial \mathbf{x}}{\partial \mathbf{X}},\quad \mathbf{v}=\dot{\mathbf{x}},\quad \mathbf{L}=\dot{\mathbf{F}}\mathbf{F}^{-1}. \tag{1}$$

Hereafter, the superscripted symbol $(\cdot)$ denotes the material time derivative of scalar or tensor fields.

To model the inelastic responses of deformed body, the standard Kröner elastic–plastic decomposition (Kröner, 1960), i.e.,

$$\mathbf{F}=\mathbf{F}^{\mathrm{e}}\mathbf{F}^{\mathrm{p}}, \tag{2}$$

is adoped here. In Eq. (2), $\mathbf{F}^{\mathrm{p}}$ is the plastic deformation gradient tensor representing the local inelastic distortion caused by the plastic mechanism; $\mathbf{F}^{\mathrm{e}}$ is the elastic deformation gradient tensor representing the subsequent stretching and rotation caused by the elastic mechanism.

In order to characterize the anisotropic inelastic responses of soft biological tissues, by referring to the work done by Ball (1977) and Ehret and Itskov (2007), (2009), a further multiplicative decomposition of the plastic deformation gradient tensor $\mathbf{F}^{\mathrm{p}}$ is newly proposed in this paper, i.e.,

$$\mathbf{F}^{\mathrm{p}}=\mathbf{F}^{\mathrm{pl}}\mathbf{F}^{\mathrm{pa}}\mathbf{F}^{\mathrm{pv}}, \tag{3}$$

where, $\mathbf{F}^{\mathrm{pv}}$,$\mathbf{F}^{\mathrm{pa}}$and $\mathbf{F}^{\mathrm{pl}}$ represent the local inelastic distortions caused by the plastic mechanisms associated with the deformations of volume, area and line elements, respectively.

According to Eqs. (1) and (3), the velocity gradient tensor$\mathbf{L}$can be rewritten as

$$\begin{aligned}\mathbf{L}=\dot{\mathbf{F}}\mathbf{F}^{-1}=&\underbrace{\dot{\mathbf{F}}^{\mathrm{e}}\mathbf{F}^{\mathrm{e}-1}}_{\mathbf{L}^{\mathrm{e}}}+\underbrace{\mathbf{F}^{\mathrm{e}}\dot{\mathbf{F}}^{\mathrm{pl}}\mathbf{F}^{\mathrm{pl}-1}\mathbf{F}^{\mathrm{e}-1}}_{\mathbf{L}^{\mathrm{epl}}}\\&+\underbrace{\mathbf{F}^{\mathrm{e}}\mathbf{F}^{\mathrm{pl}}\dot{\mathbf{F}}^{\mathrm{pa}}\mathbf{F}^{\mathrm{pa}-1}\mathbf{F}^{\mathrm{pl}-1}\mathbf{F}^{\mathrm{e}-1}}_{\mathbf{L}^{\mathrm{epa}}}+\underbrace{\mathbf{F}^{\mathrm{e}}\mathbf{F}^{\mathrm{pl}}\mathbf{F}^{\mathrm{pa}}\dot{\mathbf{F}}^{\mathrm{pv}}\mathbf{F}^{\mathrm{pv}-1}\mathbf{F}^{\mathrm{pa}-1}\mathbf{F}^{\mathrm{pl}-1}\mathbf{F}^{\mathrm{e}-1}}_{\mathbf{L}^{\mathrm{epv}}},\end{aligned} \tag{4}$$

where $\mathbf{L}^{\mathrm{e}}$ represents the elastic velocity gradient tensor; $\mathbf{L}^{\mathrm{epl}}$,$\mathbf{L}^{\mathrm{epa}}$ and $\mathbf{L}^{\mathrm{epv}}$ denote the coupled elastic–plastic velocity gradient tensors associated with the line, area and volume elements of deformed body, respectively.

Generally, the symmetric part of $\mathbf{L}$ is defined as a stretching tensor, denoted by $\mathbf{D}$, and its skew part is defined as a spin tensor, denoted by $\mathbf{W}$. It yields

$$\mathbf{L}=\mathbf{D}+\mathbf{W},\quad \mathbf{D}=\mathrm{sym}\mathbf{L}=\frac{1}{2}(\mathbf{L}+\mathbf{L}^{\mathrm{T}}),\quad \mathbf{W}=\mathrm{skw}\mathbf{L}=\frac{1}{2}(\mathbf{L}-\mathbf{L}^{\mathrm{T}}). \tag{5}$$

Apparently, from Eq. (4), the stretching tensor $\mathbf{D}$ can be additively decomposed as

$$\mathbf{D}=\mathrm{sym}\mathbf{L}^{\mathrm{e}}+\mathrm{sym}\mathbf{L}^{\mathrm{epl}}+\mathrm{sym}\mathbf{L}^{\mathrm{epa}}+\mathrm{sym}\mathbf{L}^{\mathrm{epv}}=\mathbf{D}^{\mathrm{e}}+\mathbf{D}^{\mathrm{epl}}+\mathbf{D}^{\mathrm{epa}}+\mathbf{D}^{\mathrm{epv}}, \tag{6}$$

where, $\mathbf{D}^{\mathrm{e}}$ represents the elastic stretching tensor, and $\mathbf{D}^{\mathrm{epl}}$, $\mathbf{D}^{\mathrm{epa}}$ and $\mathbf{D}^{\mathrm{epv}}$ are the coupled elastic–plastic stretching tensors related to the line, area and volume elements of deformed body, respectively.

The well-known left polar decomposition of $\mathbf{F}$ is given by

$$\mathbf{F}=\mathbf{V}\mathbf{R},\quad \mathbf{R}^{\mathrm{T}}=\mathbf{R}^{-1},\quad \mathbf{B}=\mathbf{V}^{2}=\mathbf{F}\mathbf{F}^{\mathrm{T}}, \tag{7}$$

where the orthogonal tensor $\mathbf{R}$ is a rotation tensor; while the symmetric positive definite tensors $\mathbf{B}$ and $\mathbf{V}$ are left stretch and left Cauchy–Green deformation tensors, respectively. Then, the logarithmic strain, defined as

$$\boldsymbol{\varepsilon}=\ln\mathbf{V} \tag{8}$$

is used in this work.

Similarly, the left elastic Cauchy–Green tensors is given by

$$\mathbf{B}^{\mathrm{e}}=\mathbf{F}^{\mathrm{e}}\mathbf{F}^{\mathrm{eT}}. \tag{9}$$

Obviously, $\mathbf{B}^{\mathrm{e}}$ is also a positive definite tensor.

2.2. Anisotropic free energy function

It has been shown that soft biological tissues are highly functional composite materials consisting of an isotropic matrix and several, say n, families of fibrous components (Ehret and Itskov, 2007, 2009). A unit vector $\mathbf{m}_i(i=1,2,\ldots n)$ is assumed to represent the orientation of the alignment of each fiber family. Thus $n+1$ structural tensors can be defined as

$$\mathbf{S}_i=\mathbf{m}_i\otimes\mathbf{m}_i,\quad \mathbf{S}_0=\frac{1}{3}\mathbf{1}\ \ (i=1,2,\ldots n), \tag{10}$$

with the $\mathbf{S}_0$ associated with the isotropic matrix and $\mathbf{1}$ the second-order identity tensor. Generally, setting n as 1 is often sufficient to describe the experimental results adequately (Ehret and Itskov, 2009).

Thus, a so-called generalized structural tensor is formed by the linear combinations of all the structural tensors, i.e.,

$$\mathbf{S}=\sum_{i=0}^{n}v_i\mathbf{S}_i,\quad \sum_{i=0}^{n}v_i=1 \tag{11}$$

where $v_i\geq 0\ \ (i=0,1,\ldots n)$ denote scalar weight factors. It should be noted that both the structural and generalized structural tensors are positive semi-definite tensors.

Considering the soft biological tissues to be incompressible, based on the generalized structural tensor, the generalized invariants of left elastic Cauchy–Green deformation tensor can