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Identification of artery wall stiffness: In vitro validation and in vivo results of a data assimilation procedure applied to a 3D fluid–structure interaction model

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ABSTRACT

We consider the problem of estimating the stiffness of an artery wall using a data assimilation method applied to a 3D fluid–structure interaction (FSI) model. Recalling previous works, we briefly present the FSI model, the data assimilation procedure and the segmentation algorithm. We present then two examples of the procedure using real data. First, we estimate the stiffness distribution of a silicon rubber tube from image data. Second, we present the estimation of aortic wall stiffness from real clinical data.

1. Introduction

The in vivo estimation of arterial stiffness can provide valuable information about the cardiovascular condition of a patient (Laurent et al., 2006). In clinical practice, the Pulse Wave Velocity (Nichols et al., 2005) gives an estimation of the *average* stiffness of a portion of the arterial network by solving the Moens–Korteweg equation, which assumes a linear solid in an infinite cylindrical domain. Other methods include Vascular Elastography, either based on Ultrasound (de Korte et al., 2011; Balocco et al., 2007, 2010) or Magnetic Resonance (Woodrum et al., 2006; Xu et al., 2012), where the models used for estimating arterial stiffness are again based on simplified models.

We can go further and obtain the local mechanical properties of a vessel if we adopt a data assimilation procedure, based on a three-dimensional fluid–structure interaction model and the distributed kinematic data obtained from dynamic 3D acquisitions (e.g., MRI or CT). In this way, stiffness values can be obtained under physiological conditions without requiring invasive pressure measurements.

Recently, Bertoglio et al. (2012) proposed an efficient methodology to estimate uncertain parameters (for example Young's modulus or fluid boundary conditions) from measurements of the displacement of the wall in an idealized 3D fluid–structure interaction (FSI) problem. The procedure is based on the Reduced Order Unscented Kalman Filter (ROUKF) (Moireau and Chapelle, 2011), which consists of a tractable filtering that allows to solve the estimation problem with a computational effort comparable to one forward simulation. Note that variational (in space and time) approaches have been adopted by other authors but for simplified models in Lagrée (1999), Martin et al. (2005), Stalhand (2009), and for three-dimensional problems in D'Elia et al. (2011) and Perego et al. (2011) but simplifying the minimization problem in order to avoid the resolution of the adjoint equations in time.

Whereas in Bertoglio et al. (2012) validations are only based on synthetic data, we consider in the present work the experimental validation of this method using real data in a fluid–structure interaction context. The data come from a laboratory experiment performed on a silicone tube mimicking an aorta. The results show an excellent agreement between the values obtained from data assimilation and those obtained by independent mechanical tests.

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We also apply the stiffness estimation methodology to a real clinical case, namely an aorta with a mild coarctation. Hence, this work complements the work of Moireau et al. (2012) since here we estimate the stiffness parameters and not the boundary conditions.

The rest of the paper is structured as follows. In Section 2 we present the FSI-equations, in Section 3 we summarize the ROUKF algorithm and in Section 4 we overview the image segmentation algorithm. In Section 5 we present the results for stiffness estimation with experimental data and we conclude in Section 6 with the aorta study.

2. The fluid–structure interaction equations

Mathematical model: We deal with the numerical resolution and the data assimilation of the mechanical interaction between an incompressible fluid and an elastic structure. The fluid is described by the Navier–Stokes equations (NSE), in a moving domain $\Omega^{\mathrm{f}}(t) \subset \mathbb{R}^d$, $d=2,3$, in an Arbitrary Lagrangian Eulerian (ALE) formulation, and the structure by the elastodynamic equations in $\Omega^{\mathrm{s}}(t) \subset \mathbb{R}^d$. The fluid–structure interface is denoted by $\Sigma = \partial\Omega^{\mathrm{s}} \cap \partial\Omega^{\mathrm{f}}$, and $\partial\Omega^{\mathrm{f}} = \Gamma^{\mathrm{in}} \cup \Gamma_1 \cup \cdots \cup \Gamma_{n_{0D}} \cup \Sigma$ and $\partial\Omega^{\mathrm{s}} = \Gamma^{\mathrm{n}} \cup \Gamma^{\mathrm{d}}$ are given partitions of the fluid and solid boundaries, respectively.

The coupled FSI problem reads as follows: for $t>0$, find the fluid velocity $\boldsymbol{u}_{\mathrm{f}}(t) : \Omega^{\mathrm{f}}(t) \to \mathbb{R}^d$, the fluid pressure $p(t) : \Omega^{\mathrm{f}}(t) \to \mathbb{R}$, the structure displacement $\boldsymbol{y}_{\mathrm{s}}(t) : \Omega^{\mathrm{s}}(t) \to \mathbb{R}^d$ and structure velocity $\boldsymbol{u}_{\mathrm{s}}(t) : \Omega^{\mathrm{s}}(t) \to \mathbb{R}^d$ such that

- Fluid equations:

$$\begin{cases} \rho_{\mathrm{f}} \frac{\partial \boldsymbol{u}_{\mathrm{f}}}{\partial t}\big|_{\xi} + \rho_{\mathrm{f}}(\boldsymbol{u}_{\mathrm{f}} - \boldsymbol{w}) \cdot \nabla \boldsymbol{u}_{\mathrm{f}} - \nabla \cdot \boldsymbol{\sigma}_{\mathrm{f}}(\boldsymbol{u}_{\mathrm{f}}, p) = \boldsymbol{0}, & \text{in } \Omega^{\mathrm{f}}, \\ \nabla \cdot \boldsymbol{u}_{\mathrm{f}} = 0, & \text{in } \Omega^{\mathrm{f}}, \\ \boldsymbol{u}_{\mathrm{f}} = \boldsymbol{u}_{\mathrm{in}}, & \text{on } \Gamma^{\mathrm{in}}, \\ \boldsymbol{\sigma}_{\mathrm{f}}(\boldsymbol{u}_{\mathrm{f}}, p)\boldsymbol{n}_{\mathrm{f}} = -P_i \boldsymbol{n}_{\mathrm{f}}, & \text{on } \Gamma_i, i = 1, \ldots, n_{0D}, \end{cases} \tag{1a}$$

with $\boldsymbol{\sigma}_{\mathrm{f}}(\boldsymbol{u}_{\mathrm{f}}, p) = -p\mathbf{I} + 2\mu_{\mathrm{f}}\, \boldsymbol{\varepsilon}(\boldsymbol{u}_{\mathrm{f}})$, the Cauchy stress tensor of the fluid and $\boldsymbol{\varepsilon}(\boldsymbol{u}_{\mathrm{f}}) = (\nabla \boldsymbol{u}_{\mathrm{f}} + (\nabla \boldsymbol{u}_{\mathrm{f}})^{\mathrm{T}})/2$ denotes the deformation rate tensor, μ_{f} the dynamic viscosity, and $\partial/\partial t|_{\xi}$ the ALE derivative (see e.g. Fernández and Gerbeau, 2009). In the hemodynamics problems considered in this work, the outlet pressure P_i is obtained by solving the differential-algebraic equation, known as the Windkessel model (Peiró and Veneziani, 2009)

$$\begin{cases} P_i = \pi_i + R_{p,i} Q_i, \\ C_i \dfrac{d\pi_i}{dt} + \dfrac{\pi_i}{R_{d,i}} = Q_i, \quad Q_i = \int_{\Gamma_i} \boldsymbol{u}_{\mathrm{f}} \cdot \boldsymbol{n}_{\mathrm{f}}. \end{cases} \tag{1b}$$

The distal resistance $R_{d,i}$, the proximal resistance $R_{p,i}$ and the capacitance C_i are assumed to be given.

- Structure equations:

$$\begin{cases} \partial_t \boldsymbol{y}_{\mathrm{s}} = \boldsymbol{u}_{\mathrm{s}}, & \text{in } \Omega^{\mathrm{s}}, \\ \rho_{\mathrm{s}} \partial_t \boldsymbol{u}_{\mathrm{s}} - \eta_{\mathrm{s}} \nabla \cdot \boldsymbol{\sigma}_{\mathrm{s}}(\boldsymbol{u}_{\mathrm{s}}) - \nabla \cdot \boldsymbol{\sigma}_{\mathrm{s}}(\boldsymbol{y}_{\mathrm{s}}, \theta) = \boldsymbol{0}, & \text{in } \Omega^{\mathrm{s}}, \\ \boldsymbol{y}_{\mathrm{s}} = \boldsymbol{y}_{\mathrm{in}}, & \text{on } \Gamma^{\mathrm{d}}, \\ \eta_{\mathrm{s}} \boldsymbol{\sigma}_{\mathrm{s}}(\boldsymbol{u}_{\mathrm{s}})\boldsymbol{n}_{\mathrm{s}} + \boldsymbol{\sigma}_{\mathrm{s}}(\boldsymbol{y}_{\mathrm{s}})\boldsymbol{n}_{\mathrm{s}} = -c_{\Gamma} \boldsymbol{u}_{\mathrm{s}} - k_{\Gamma} \boldsymbol{y}_{\mathrm{s}}, & \text{on } \Gamma^{\mathrm{n}}, \end{cases} \tag{1c}$$

where $\boldsymbol{\sigma}_{\mathrm{s}}$ is the Cauchy stress tensor of the solid and the vector $\theta \in \mathbb{R}^{\kappa}$ denotes the set of solid constitutive parameters that will be estimated later. In this work, both quantities are related by

  - Considering a linear constitutive relation, namely $\boldsymbol{\sigma}_s(\boldsymbol{y}_{\mathrm{s}}, \theta) = \mathbf{C}(\theta)\boldsymbol{\epsilon}(\boldsymbol{y}_{\mathrm{s}})$ with $\boldsymbol{\epsilon}(\boldsymbol{y}_{\mathrm{s}}) = (\nabla \boldsymbol{y}_{\mathrm{s}} + (\nabla \boldsymbol{y}_{\mathrm{s}})^{\mathrm{T}})/2$ and $\mathbf{C}$ the classical elasticity tensor, where the Young's modulus is given by $E = E_0 \cdot 2^{\theta}$, $E_0 > 0$, while the Poisson's ratio is fixed.
  - Considering a nonlinear, hyperelastic, Neo-Hookean material model, such that the elastic energy density function is given by $\mathcal{W} = c_0 \cdot 2^{\theta} \cdot (I_1 - 3)$, $c_0 > 0$ and I_1 the first invariant of the right Cauchy-Green strain tensor.

Moreover, the parameters c_{Γ} and k_{Γ} model in a simple way the external tissue effect on the vessel of interest (Moireau et al., 2011, 2012) and η_{s} is a viscoelastic coefficient.

- Coupling conditions:

$$\begin{cases} \boldsymbol{y}_{\mathrm{f}} = \mathrm{Ext}^{\mathrm{f}}_{\Sigma(0)}(\boldsymbol{y}_{\mathrm{s}|_{\Sigma}}), \quad \boldsymbol{w} = \partial_t \boldsymbol{y}_{\mathrm{f}}, \quad \Omega^{\mathrm{f}}(t) = (I_{\Omega^{\mathrm{f}}} + \boldsymbol{y}_{\mathrm{f}}(t))(\Omega^{\mathrm{f}}), & \\ \boldsymbol{u}_{\mathrm{f}} = \boldsymbol{u}_{\mathrm{s}}, & \text{on } \Sigma, \\ \eta_{\mathrm{s}} \boldsymbol{\sigma}_{\mathrm{s}}(\boldsymbol{u}_{\mathrm{s}})\boldsymbol{n}_{\mathrm{s}} + \boldsymbol{\sigma}_{\mathrm{s}}(\boldsymbol{y}_{\mathrm{s}})\boldsymbol{n}_{\mathrm{s}} + \boldsymbol{\sigma}_{\mathrm{f}}(\boldsymbol{u}_{\mathrm{f}}, p)\boldsymbol{n}_{\mathrm{f}} = \boldsymbol{0}, & \text{on } \Sigma, \end{cases} \tag{1d}$$

where $\mathrm{Ext}^{\mathrm{f}}_{\Sigma}$ is an extension operator from Σ to Ω^{f}.

FSI numerical algorithm: The spatial discretization is performed with a first order stabilized finite element. For the time marching, we use a semi-implicit partitioned FSI-algorithm (Fernández et al., 2006, 2007) with a first order Chorin–Temam projection method in the fluid, and a Newmark scheme in the solid. The coupling between the solid and the fluid is explicit in the ALE-advection-diffusion step and implicit for the pressure projection step and the Windkessel (Bertoglio et al., 2013a).

3. The data assimilation procedure

In this section we summarize the data assimilation procedure based on Moireau and Chapelle (2011) and already studied in Bertoglio et al. (2012) for idealized (but realistic) three-dimensional arterial FSI problems.

Inverse problem statement: Assume that we can write the fully discrete FSI-problem as

$$X_{n+1} = \mathcal{A}_n(X_n, \theta), \quad n \geq 0, \tag{2}$$

with $X_n = (\boldsymbol{u}_{\mathrm{f}}^n, \boldsymbol{y}_{\mathrm{s}}^n, \boldsymbol{u}_{\mathrm{s}}^n) \in \mathbb{R}^q$ the discrete dynamical state, $\theta \in \mathbb{R}^{\kappa}$ the physical parameters, and X_0 the given initial condition. Suppose that θ and X_0 are uncertain, and that we have (noised) measurements of real physical system $Z_n \in \mathbb{R}^z$ related to the numerical state by the observation error $\Gamma_n(Z_n, X_n) = \zeta_n$, with $\zeta_n \in \mathbb{R}^m$ the noise. Data assimilation consists in reducing the uncertainties of a model by minimizing a cost function like (see, e.g., Banks and Kunisch, 1989):

$$J(X_0, \theta_0) = \sum_{n=0}^{N} \| \Gamma_n(Z_n, X_n) \|^2_{W_n^{-1}} + \| \theta_0 - \hat{\theta}_0^+ \|^2_{(P_0^{\theta})^{-1}} + \| X_0 - \hat{X}_0^+ \|^2_{(P_0^X)^{-1}},$$

with X_n satisfying (2). In this expression, $\hat{X}_0$ and $\hat{\theta}_0$ are given *a priori* values and $\| \cdot \|_{W^{-1}}$, $\| \cdot \|_{(P_0^X)^{-1}}$ and $\| \cdot \|_{(P_0^{\theta})^{-1}}$ denote some matrix norms used to give a different weight to the different terms.

This minimization problem can be addressed by many methods that are classically divided into two groups: the variational and the sequential approaches. Variational approaches minimize this cost function by an optimization algorithm – usually a gradient-based computed from the solution of an adjoint model – and require numerous computations of the forward – and possibly the adjoint – problem, see e.g. Martin et al. (2005). Here, we follow a sequential, or *filtering*, approach which modifies the forward dynamics (2) with a correction term proportional to the observation error. More precisely, we adopt the Reduced-Order Unscented Kalman Filter (ROUKF) (Moireau and Chapelle, 2011), inspired from Julier et al. (1995, 2000). It does not require any tangent operator and allows one to run the estimation with a computational cost of the same order of the forward problem since it is highly parallelizable. Moreover, it requires only superficial modifications to the existing solvers. Its main assumption consists of neglecting the uncertainty in X_0. Hence $P_0^X \to \infty$ and a reduced-order form of the Kalman filter can be obtained. Note that it is also possible to take into account the uncertainties in the initial condition in the same framework with an additive nudging based correction on the dynamics as introduced