Journal of Biomechanics 46 (2013) 1921–1927

Contents lists available at SciVerse ScienceDirect

Journal of Biomechanics

journal homepage: www.elsevier.com/locate/jbiomech
www.JBiomech.com

Short communication

Using regression models to determine the poroelastic properties of cartilage

Chen-Yuan Chung, Joseph M. Mansour *

Department of Mechanical and Aerospace Engineering, Case Western Reserve University, 2123 Martin Luther King Jr. Drive, Cleveland, OH 44106, USA

ARTICLE INFO

Article history:
Accepted 26 May 2013

Keywords:
Stress relaxation
Poroelasticity
Transversely isotropic
Linear regression

ABSTRACT

The feasibility of determining biphasic material properties using regression models was investigated. A transversely isotropic poroelastic finite element model of stress relaxation was developed and validated against known results. This model was then used to simulate load intensity for a wide range of material properties. Linear regression equations for load intensity as a function of the five independent material properties were then developed for nine time points (131, 205, 304, 390, 500, 619, 700, 800, and 1000 s) during relaxation. These equations illustrate the effect of individual material property on the stress in the time history. The equations at the first four time points, as well as one at a later time (five equations) could be solved for the five unknown material properties given computed values of the load intensity. Results showed that four of the five material properties could be estimated from the regression equations to within 9% of the values used in simulation if time points up to 1000 s are included in the set of equations. However, reasonable estimates of the out of plane Poisson's ratio could not be found. Although all regression equations depended on permeability, suggesting that true equilibrium was not realized at 1000 s of simulation, it was possible to estimate material properties to within 10% of the expected values using equations that included data up to 800 s. This suggests that credible estimates of most material properties can be obtained from tests that are not run to equilibrium, which is typically several thousand seconds.

1. Introduction

Mechanically, articular cartilage is often modeled as a two-phase material with solid and fluid phases. Parallel constitutive models have been proposed using either biphasic (Mow et al., 1980) or poroelastic (Simon et al., 1983) theories. Although these theories developed from different roots, the governing equations of linear biphasic theory are mathematically equivalent to those of Biot's theory (Biot, 1941) for linear quasi-static poroelasticity with incompressible constituents (Simon, 1992). Using these models, and tests such as confined or unconfined compression, constitutive constants can be determined by fitting analytical models to measured data (Armstrong et al., 1984; Bursać et al., 1999; Mow et al., 1980). Typically, material properties are found using an optimization procedure that fits a model to experimental data (Lei and Szeri, 2007; Cao et al., 2006; Athanasiou et al., 1995).

The objective of this work is to investigate the feasibility of using regression models to determine the poroelastic properties of cartilage tested in unconfined compression. Such models could simplify the process for determining material properties from measured data.

2. Methods

An axisymmetric poroelastic model for unconfined compression stress relaxation was developed using ANSYS. In an unconfined compression stress relaxation test, a thin cylindrical specimen is compressed between two rigid impermeable and smooth platens while surrounded by fluid (Fig. 1).

For this investigation we used a transversely isotropic model that shows good agreement with unconfined compression measurements in growth plate (Cohen et al., 1998). The compliance matrix for transverse isotropy has five independent constants (Bower, 2009)

$$S_{ijkl} = \begin{bmatrix} \frac{1}{E_t} & -\frac{\nu_t}{E_t} & -\frac{\nu_{at}}{E_a} & 0 & 0 & 0 \\ -\frac{\nu_t}{E_t} & \frac{1}{E_t} & -\frac{\nu_{at}}{E_a} & 0 & 0 & 0 \\ -\frac{\nu_{at}}{E_a} & -\frac{\nu_{at}}{E_a} & \frac{1}{E_a} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\mu_a} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\mu_a} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu_t)}{E_t} \end{bmatrix} \quad (1)$$

where E, ν, and μ are Young's modulus, Poisson's ratio, and shear modulus, respectively. The subscripts t and a indicate the transverse and axial or out-of-plane directions.

Cartilage was modeled using the coupled pore-pressure element CPT213, which is based on Biot's poroelasticity theory (ANSYS, 2010). This element is a fully direct

* Corresponding author. Tel.: +1 2163684190; fax: +1 2163683007.
E-mail addresses: mansour@case.edu, jmm12@case.edu (J.M. Mansour).

0021-9290/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jbiomech.2013.05.028

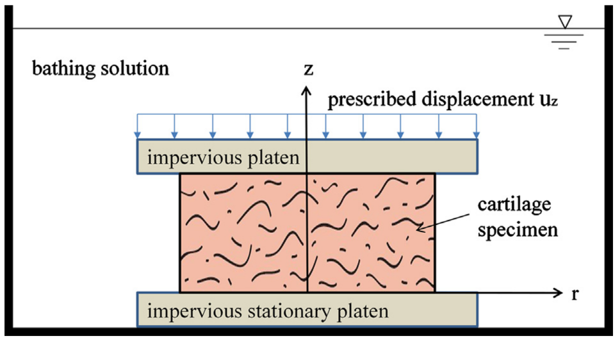

Fig. 1. Schematic of the unconfined compression test of a cylindrical disk of hydrated cartilage. The same boundary conditions used in Cohen et al. (1998), were applied. The specimen was free to expand in the radial direction. Displacements in the axial direction were constrained on the bottom of the specimen in contact with the stationary platen. Free fluid flow was enabled across the lateral boundaries, i.e., fluid pressure was set as zero on the cylindrical periphery. In contrast, the fluid was not permitted across the boundaries with the upper and lower platens.

Table 1
Values of material properties used to simulate stress relaxation in the transversely isotropic finite element model.

Material property	Range
Young's modulus in plane E_t (MPa)	4.3–10.3 in 0.5 increment
Young's modulus out of plane E_a (MPa)	0.3–1.1 in 0.1 increment
Poisson's ratio in transverse plane ν_t	0.24–0.49 in 0.05 increment
Poisson's ratio out of plane ν_{at}	0–0.1 in 0.02 increment
Permeability k ($\times 10^{-15}$ m^4 N^{-1} s^{-1})	1.8–5.0 in 0.4 increment

coupling in which the mechanical equilibrium and fluid continuity equations are satisfied simultaneously. The axis of symmetry coincided with the global Y-axis in ANSYS. The directions of X, Y and Z were radial, axial and circumferential, respectively. The stress in the Y direction in ANSYS represents the effective stress in the Z direction in Armstrong et al. (1984). The total compressive stress is equal to the fluid pressure subtracted from the effective stress, i.e., $\sigma_a^t = \sigma_a^e - p_0$. Using ANSYS, transverse isotropy is modeled as a special case of orthotropic isotropy. Although nine elastic constants are needed to model orthotropic isotropy, only five independent constants are needed for transverse isotropy.

We applied the boundary conditions used by Cohen et al. (1998): the displacement imposed on the specimen was linear over time, reached its maximum value of 10% of the cartilage thickness at $t_0 = 131$ s. It was then held constant for 869 s. Axial displacement was constrained on the bottom of the specimen. Free fluid flow was enabled across the lateral boundaries. In contrast, the fluid was not permitted across the boundaries with the upper and lower platens.

The computational model was validated using data and results in Cohen et al. (1998). The specimen was a cylinder with radius $a = 3.175$ mm and thickness $h = 1$ mm. The five independent elastic constants were Young's moduli in the transverse plane $E_t = 4.3$MPa and out of plane $E_a = 0.64$MPa, Poisson's ratios in the transverse plane $\nu_t = 0.49$ and out of plane $\nu_{at} = 0$ and the out-of-plane shear modulus, μ_a, which could be any value for the case of uniaxial loading. Permeability was assumed to be the same in both the axial and transverse directions: $k = 5 \times 10^{-15}$ m^4 N^{-1} s^{-1}. Based on the assumption that, for a soft tissue the solid and fluid phases are incompressible, the Biot coefficient equals one, and the reciprocal of the Biot modulus is zero (Simon, 1992). Load intensity at each time was computed by dividing the total force on the top surface of the model by its undeformed area. Nonlinear static analysis was performed using the Newton-Raphson algorithm with an unsymmetric option. A macro, written in the ANSYS parametric design language, performed the procedure automatically.

The load intensity (f) was computed at nine time points during relaxation: 131 s (when the ramp displacement reached the maximum) and 205, 304, 390, 500, 619, 700, 800 and 1000 s. These points were chosen from among the time points in Cohen et al. (1998). Simulations were performed using a range of material properties of growth plate (Villemure and Stokes, 2009) (Table 1). Two of the five properties were varied in each set of simulations, resulting in ten combinations of properties. Within each combination, simulations were performed for the range of material properties in Table 1. This process systematically populated the solution space for load intensity as a function of feasible material properties. Load intensity was normalized ($\bar{f}$) by the maximum value obtained from all simulations at a given time. Each material property was normalized by its largest value (Table 1).

Regression models were obtained by fitting the normalized load intensity to first-order polynomial equations in the normalized material properties at each time point, which resulted in nine equations. Any five of these equations can be solved

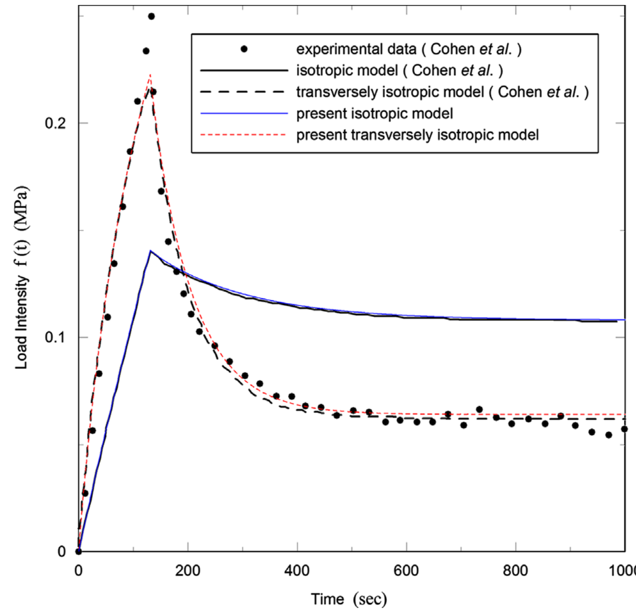

Fig. 2. Comparison of the transversely isotropic and an isotropic model, both developed using ANSYS, and the analytical solution of those models and experimental results from Cohen et al. (1998). For the isotropic model, Young's modulus was $E = 1.08$ MPa, Poisson's ratio was $\nu = 0$, and permeability was $k = 15.5 \times 10^{-15}$ m^4 N^{-1} s^{-1}, which are the same values used in Cohen et al. (1998).

using known values of the load intensity to obtain estimates of the five unknown material properties. In this investigation, known load intensity was obtained from simulated stress relaxation found using the best-fit material property data from Cohen et al. (1998). Material properties obtained from the regression equations and reverted to physical values (p_r) were compared with those used to validate our computational model (p_v). The error in the predicted properties was computed using error $= |(p_r - p_v)/p_v| \times 100\%$.

3. Results

Load intensity obtained from our finite element model is almost identical to that given by Cohen et al. (1998), which validates the computational model (Figs. 2 and 3). Although load intensity varied approximately linearly with material properties, there were regions with nonlinear behavior (Figs. 4–13). For example, load intensity at 1000 s deviated from linear behavior for E_t between 4.3 MPa and 6 MPa and for ν_t between 0.24 and 0.4 (Fig. 6b). Load intensity at 1000 s also deviated from linear behavior for E_t between 4.3 MPa and 6 MPa and for k between 1.8×10^{-15} m^4 N^{-1} s^{-1} and 3×10^{-15} m^4 N^{-1} s^{-1} (Fig. 8b).

Linear regression equations for the normalized load intensity at specific times (Eqs. (2)–(10)) showed good fits to the simulated data ($p < 0.001$ and $R^2 > 0.9$ for all equations). However, the coefficients of ν_{at} at 304, 390 and 500 s, shown in the equations below, were not significant ($p > 0.05$) and were set to zero when using the corresponding regression equations to determine mechanical properties.

$$\bar{f}_{131\,s} = 0.4874 + 0.2054\bar{E}_t + 0.2250\bar{E}_a + 0.1032\bar{\nu}_t - 0.0659\bar{\nu}_{at} - 0.3534\bar{k} \quad (2)$$

$$\bar{f}_{205\,s} = 0.9387 - 0.2107\bar{E}_t + 0.3950\bar{E}_a - 0.0756\bar{\nu}_t - 0.0422\bar{\nu}_{at} - 0.5852\bar{k} \quad (3)$$

$$\bar{f}_{304\,s} = 0.7281 - 0.1787\bar{E}_t + 0.5179\bar{E}_a - 0.1121\bar{\nu}_t - 0.0082\bar{\nu}_{at} - 0.4822\bar{k} \quad (4)$$

$$\bar{f}_{390\,s} = 0.5227 - 0.1114\bar{E}_t + 0.6175\bar{E}_a - 0.0960\bar{\nu}_t + 0.0030\bar{\nu}_{at} - 0.3628\bar{k} \quad (5)$$