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Short communication

Harmonic ratios: A quantification of step to step symmetry

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ABSTRACT

The harmonic ratio (HR), derived from the Fourier analysis of trunk accelerations, has been described in various ways as a measure of walking smoothness, walking rhythmicity, or dynamic stability. There is an increasing interest in applying the HR technique to investigate the impact of various pathologies on locomotion; however, explanation of the method has been limited. The aim here is to present a clear description of the mathematical basis of HRs and an understanding of their interpretation. We present harmonic theory, the interpretation of the HR using sinusoidal signals, and an example using actual trunk accelerations and harmonic analyses during limb-loading conditions. We suggest that the HR method may be better defined, not as a measure of rhythmicity or stability, but as a measure of step-to-step symmetry within a stride.

1. Introduction

The harmonic ratio (HR) is a measure used to quantify smoothness of walking (Gage, 1964; Menz et al., 2003b; Smidt et al., 1971; Yack and Berger, 1993). In gait research, the HR is most commonly extracted from trunk accelerations in the antero-posterior (AP), vertical (VT) and mediolateral (ML) directions. The VT and AP accelerations are biphasic within a stride due to the right and left steps, while the ML accelerations are monophasic (Fig. 1). The HR quantifies the harmonic composition of these accelerations for a given stride, where a high HR is interpreted as greater walking smoothness. The HR calculated from trunk accelerations, unlike typical spatiotemporal parameters, is a summary measure of whole body movement. HRs have discriminated between the gait of young and older adults (Brach et al., 2011; Kavanagh et al., 2005a; Yack and Berger, 1993), older adults who have and have not fallen (Menz et al., 2003a), and the gait of healthy older adults and individuals with neurologic disorders (Latt et al., 2009; Lowry et al., 2009; Menz et al., 2004). Recently there is an increasing interest in applying the HR technique to discriminate between individuals with different pathologies and to monitor the impact of rehabilitation protocols.

While showing potential as a gait analysis tool, the terminology used to define and explain HRs has generated some confusion across disciplines. An early study described HRs as ‘‘reflecting the frequency of force changes experienced by the body during the walking cycle, providing an index for the smoothness of walking’’ (Smidt et al., 1977). However, researchers have also used the terms *rhythmicity* (Latt et al., 2008; Menz et al., 2003b), *dynamic stability* (Yack and Berger, 1993), and *walking balance* (Menz et al., 2003c) interchangeably with smoothness of walking. These terms do not share the same meaning. Rhythmicity can refer to any rhythm, whereas HRs specifically quantifies the biphasic and monophasic natures of the signals. Also, dynamic stability has been recently defined by researchers as ‘‘the sensitivity of a system to infinitesimally small perturbations’’ and been quantified by Lyapunov exponents (Dingwell and Marin, 2006). Our goal is to provide a clear description of the mathematical basis of HRs and an understanding of their physiological interpretation.

2. Methods

2.1. Harmonic theory

The HR method is based upon harmonic theory to examine the symmetry within a stride by exploiting the periodicity of the signal (Gage, 1964; Smidt et al., 1971). The measured accelerations for each stride are analyzed in the frequency domain through a well-established technique of Fourier analysis based on the stride frequency (Oppenheim et al., 1997). Since the DFT is performed on each stride separately, the measure allows for variance of the stride frequency within a trial. However, given the basis in single stride analysis, the measure is sensitive to errors in stride frequency; thus, consistency of stride segmentation location (i.e. identification of heel contact) is very important. The HR is then defined from the DFT as a ratio of the sum of the amplitudes of the even harmonics ($n=2,4,6,\ldots$) to the sum of the amplitudes of the odd harmonics ($n=1,3,5,\ldots$). However,

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because of the differing biphasic and monophasic natures of the AP and VT versus ML components, the ratios of the odd to even harmonics are calculated inversely.

Traditionally, the first 20 harmonic coefficients are used to calculate the HRs (Smidt et al., 1971). This is justified for normal cadences, because the majority of the power occurs below 10 Hz (Kavanagh et al., 2005b), and the normal stride frequency generally ranges from 0.8 Hz for older adults walking slowly to 1.1 Hz for young adults walking quickly (100–135 steps/min respectively) (Hollman et al., 2011; Sekiya and Nagasaki, 1998). Cadences of 60 steps/min or slower (0.5 Hz stride frequency) may be problematic, since the first 20 harmonics only encompasses frequencies up to 10 Hz. When using HRs, the power spectrum should be inspected to ensure enough coefficients are incorporated into the ratio, and the number of coefficients can be increased by 2's as needed.

2.2. Sinusoid example

To demonstrate the differing effects of even and odd harmonic components, we present a simple example implementing the HR technique on sinusoids for the biphasic case. Two harmonics of a 1 Hz sinusoid (2nd and 3rd harmonics) are used. The 1 Hz sinusoid can be thought of as a "stride frequency," and the "steps" are half of the sinusoid, bisected at 0.5 s. In Fig. 2 we depict the even 2nd harmonic, the odd 3rd harmonic, and a summation of scaled 2nd and 3rd harmonics. As odd harmonics cannot be divided evenly within each step, adding them to the even harmonics results in differing constructive and destructive interference in the first and second halves of the cycle (i.e. each step). Thus, interference between the amplitudes of the odd and even harmonics results in changes in magnitude and timing between steps; this effect is entirely based on the magnitude of the harmonic frequencies and not the phase. This effect is reflected in the HR, as in this sinusoidal example, the HR for the 2nd harmonic alone would be infinity and the summation of the 2nd and 3rd harmonics would be reduced to 2.5.

The phases of the even and odd harmonics within a stride are not used within the calculation of the HR. The unaccounted-for phase of each harmonic effects where in the stride the magnitude and timing changes occur, but not the amount. Thus, an infinite number of phase combinations can produce the same HR. Fig. 3 displays an example of the effect of phase shift, where one of the summations of 2nd and 3rd harmonics has a 45° phase shift added to the 3rd harmonic. The phase shift modulates the shape of the signal but not the amount of magnitude and timing changes and both summations have the same HR.

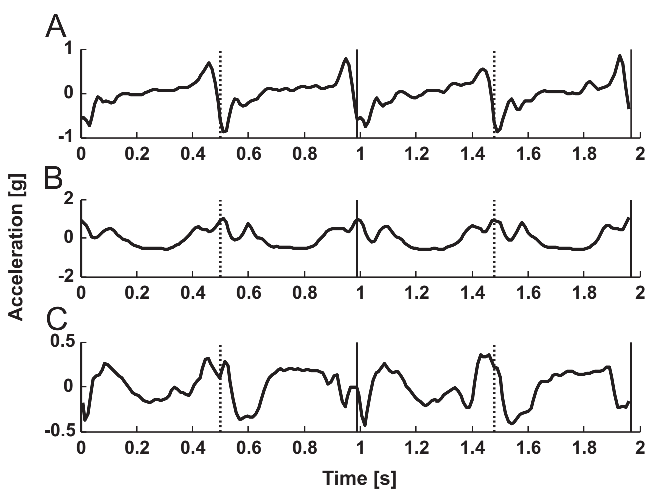

Fig. 1. The L3/L4 acceleration signal for (A) anterioposter, (B) vertical and (C) mediolateral directions with right (vertical solid) and left (vertical dotted) heel contacts. *Note*: the gravitational component has been removed.

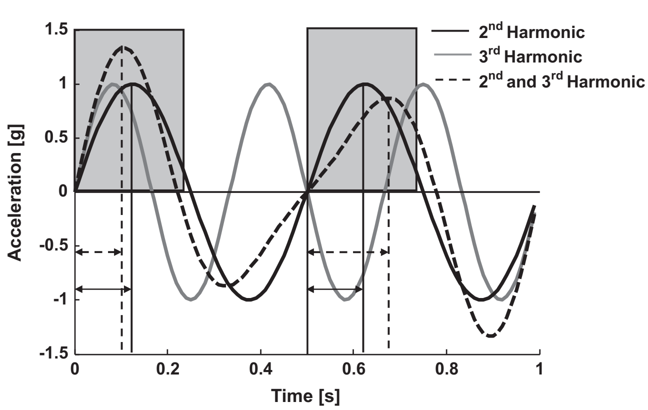

Fig. 2. Effect of odd harmonic components on sinusoidal summations. When compared to the regular patterns of the 2nd (black solid line; $\sin(2\times2\pi)$) and the 3rd harmonics (gray solid line; $\sin(3\times2\pi)$), the summation of the 2nd and 3rd (dashed line; $\sin(2\times2\pi)+0.4\ \sin(3\times2\pi)$) demonstrates changes in both timing and magnitude between steps. The 3rd harmonic constructively combines with the positive peak of the 2nd harmonic in the first step and destructively in the second, generating the magnitude differences in the summation highlighted by the gray regions. The constructive/destructive summation also induces a timing shift in the peaks as indicated by the dashed vertical lines from the start of each step, compared to the consistent solid vertical lines of the 2nd harmonic.

2.3. Example using unilateral limb loading data

To demonstrate the ability of HR analysis to identify asymmetry between steps, we used a unilateral limb loading paradigm. Adding weight to one leg has been shown to induce asymmetry in both kinetics and kinematics (Haddad et al., 2006; Kodesh et al., 2012; Smith and Martin, 2007). Thus, the HR should decrease as the asymmetry increases with increased loading.

The subject walked 9 m along a straight path. AP, VT, and ML accelerations were collected from an accelerometer attached posteriorly at L3/L4. Data collected during the middle 7.5 m were used. The subject walked to the beat of a metronome to maintain consistent cadence and was instructed to take the same number of steps to maintain gait speed. A unilateral load was applied via cuff weights secured around the right ankle (masses of 0, 0.45, 1.36, and 2.27 kg) (Kodesh et al., 2012). Identification of heel contact for stride segmentation was determined using footswitches. HRs were calculated for each stride and averaged across two trials (Brach et al., 2011).

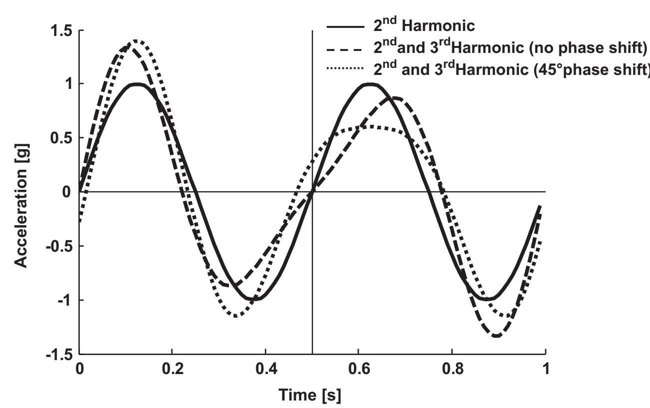

Fig. 3. The effect of phase shift on sinusoidal summations. The summation of the 2nd and 3rd harmonics with zero phase (dashed line; $\sin(2\times2\pi)+0.4\ \sin(3\times2\pi)$) and the summation of the 2nd and 3rd harmonics with 45° of phase shift added to the 3rd harmonic (dotted line; $\sin(2\times2\pi)+0.4\ \sin(3\times2\pi+\pi/4)$) are shaped differently. However, the timing and magnitude changes between steps caused by the 3rd harmonic can still be seen when comparing to the 2nd harmonic alone (solid line; $\sin(2\times2\pi)$). The phase shift only changes the location of these changes, hence the harmonic ratio of these signals remain the same (2.5 summations; ∞ 2nd harmonic only).

Table 1
Harmonic ratios for the limb loading conditions.

Condition (kg)	Harmonic ratio		
	AP	VT	ML
0	3.55	5.16	2.67
0.45	3.68	4.18	2.21
1.36	2.11	2.46	1.82
2.27	1.69	1.88	1.49