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Viscoelastic properties of gerbil tympanic membrane at very low frequencies

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ABSTRACT

The tympanic membrane transfers sound waves in the ear canal to mechanical vibrations in the middle ear and cochlea. Good estimates of the mechanical properties of the tympanic membrane are important to obtain realistic models. Up till now, only limited resources about tympanic membrane viscoelastic properties are available in the literature.

This study aimed to quantify the viscoelastic properties of gerbil tympanic membrane. Step indentations were applied with a custom indenter on four fresh, intact tympanic membranes and the resulting force relaxation was measured. The reduced relaxation functions were then fitted with two viscoelastic model representations: a 5-parameter Maxwell model and a model with a continuous relaxation spectrum.

The average relaxation function is described by an initial rapid decrease of 6.5% with characteristic time 0.77 s, followed by a long term decrease with characteristic time 46 s that gradually tends stable till a total relaxation of 15%. The relaxation curves in the time domain were transformed to complex moduli in the frequency domain. It was found that these transformations yield information on strain-rate dependence only from quasi-static to the very lowest acoustic frequencies.

Finally, relaxation and hysteresis were simulated in a finite element model with viscoelastic material properties.

1. Introduction

The lack of high-quality estimates of tympanic membrane mechanical properties has been a bottleneck in the refinement of middle ear finite element models for years. We recently published a paper that contributes to work in this area: Aernouts and Dirckx (in press). In that paper, we describe in situ indentation measurements on gerbil tympanic membrane using a custom built setup. Both quasi-static and dynamic sinusoidal indentation loadings were applied. Due to experimental limitations, however, the studied frequency domain was limited from 0.2 to 8.2 Hz. The experiments were simulated with a finite element model and Young's modulus and Poisson's ratio of the tympanic membrane were optimized in order to fit the experimental data. The frequency dependence of Young's modulus was then described with two viscoelastic representations, a standard linear solid and a model with a continuous relaxation spectrum.

The present paper is a follow-up to Aernouts and Dirckx (in press). We now show results of stress relaxation tests by applying step indentations. Stress relaxation tests are a typical way to investigate viscoelastic properties of biological tissue, e.g. for ankle ligaments (Funk et al., 2000) and for subcutaneous tissue (Iatridis et al., 2003).

Measurements were performed on the same samples at the same time as those described in the first paper. The experimental results are again described using viscoelastic representations and the viscoelastic parameters are used to simulate tympanic membrane stress relaxation and hysteresis in a finite element model.

Finally, time relaxation data are converted to the frequency domain and compared with the earlier work. The current results extend the data of the previous work to lower frequencies of the order 10^{-4} Hz. The relaxation data, however, do not contain strain rate dependence for higher frequencies.

For more background on middle ear finite element modelling and previous work on tympanic membrane mechanical properties, we refer to Aernouts and Dirckx (in press).

2. Methods

2.1. Specimen preparation

Testing was done on fresh Mongolian gerbil samples (*Meriones unguiculatus*) following the rules of the local ethical committee. Middle ear cavities were dissected, the tympanic membrane was made accessible from the medial side and the ossicular chain was immobilized by gluing the malleus to the middle ear wall. Experiments were performed on the same samples as in the first paper

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(Fig. 2 in Aernouts and Dirckx, in press), right after the measurements therein mentioned were finished.

During sample preparation and indentation testing, the specimens were kept moist with an ultrasonic humidifier (Bionaire). Sample preparation and testing took less than 75 min.

2.2. Stress relaxation tests

Specimens were placed in a custom built indentation setup. We used the same setup as before (Fig. 3 in Aernouts and Dirckx, in press), however it was now used to apply needle step displacements.

Each sample was first preconditioned by 10 cycles of 40 μm oscillatory displacement at 1 Hz, followed by a 120 s wait. The use and type of preconditioning might affect the results. In Funk et al. (2000), the effect of preconditioning on viscoelastic behavior of ligaments was studied. It was found that preconditioning only affects the short time behavior: a higher peak force value was observed for the unpreconditioned state; however, ligament specimens eventually relaxed to the same force level regardless of whether they were in a preconditioned state or not. Furthermore, the time constants were found to be similar regardless of the state of preconditioning.

After preconditioning, a step displacement was applied and the resulting force relaxation was measured during 120 s at a sample rate of 10 000 samples/s. The needle was positioned locally perpendicular to avoid slippage. We used indentation depths of 40 or 60 μm. In Fig. 1, the actual needle displacement is plotted for a 60 μm step. The rise time, measured between the two dashed vertical lines, is 1.2 ms; there is an overshoot of 1.7 μm. Repeatability of this step waveform was observed throughout measurements.

The recorded force relaxation curves showed a significant noise level. To reduce this, the data were smoothed with a locally weighted scatterplot smoothing using the *smooth* command in Matlab. It was assured that this did not contaminate the data; see the results section. From the smoothed force relaxation curves, the reduced relaxation functions were obtained by normalizing the relaxation data by the peak force values.

2.3. Viscoelastic models

The reduced relaxation functions were fitted with two viscoelastic models: a 5-parameter Maxwell model and a continuous relaxation model.

2.3.1. Generalized Maxwell model

In our previous paper, we first used the standard linear solid model to fit the complex modulus in the frequency domain. The standard linear solid model is composed of a spring R_0 in parallel with a Maxwell model (a spring R_1 and dashpot η_1 in series). From the standard linear solid differential equation (see Eq. (9) in Aernouts and Dirckx, in press), the stress response to a step strain can be calculated by subjecting a constant strain ε_0 at $t=0$. The solution is

$$\sigma(t) = R_1\varepsilon_0 e^{-t/\tau_1} + R_0\varepsilon_0 \tag{1}$$

with $\tau_1 = \eta_1/R_1$. Dividing by $\sigma_0 = R_1\varepsilon_0 + R_0\varepsilon_0$ gives the reduced relaxation function for the standard linear solid

$$G(t) = g_1 e^{-t/\tau_1} + g_\infty \tag{2}$$

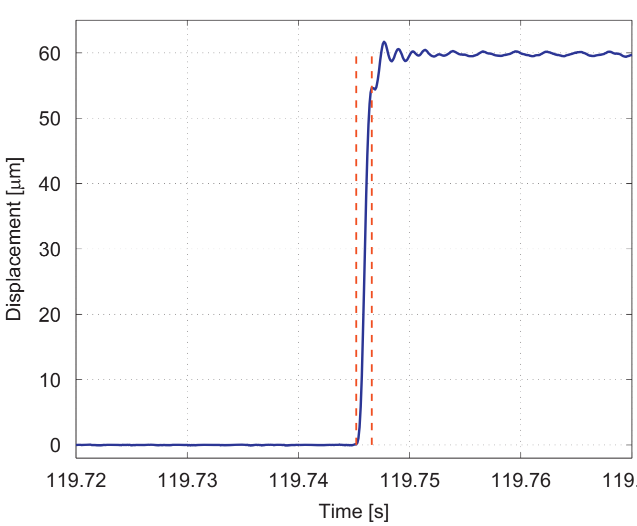

Fig. 1. Actual displacement for a 60 μm applied step. The rise time (time interval between the vertical dashed lines) is 1.2 ms.

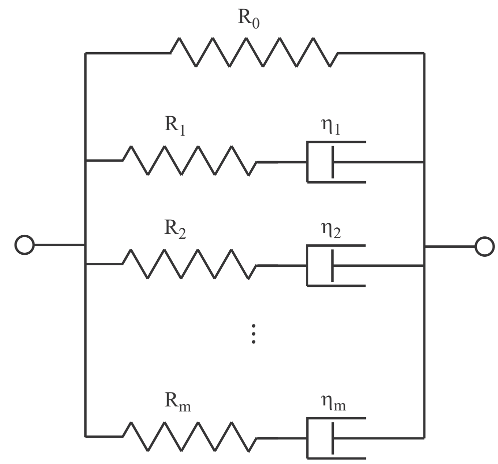

Fig. 2. Model representation for the generalized Maxwell model.

with constants g_1, τ_1 and g_∞ and for which holds $G(0)=1$. This equation has only one exponential.

The generalized Maxwell model is obtained by putting more Maxwell elements in parallel with the standard linear solid, as shown in Fig. 2. By putting more branches in parallel, more exponentials are added to the relaxation function (e.g. Zhang et al., 2007). We found that two exponentials (five parameters) were sufficient to produce a good fit; see the results section. The reduced relaxation function for the 5-parameter Maxwell model is given by

$$G(t) = g_1 e^{-t/\tau_1} + g_2 e^{-t/\tau_2} + g_\infty \tag{3}$$

with constants g_1, τ_1, g_2, τ_2 and g_∞ and for which holds $G(0)=1$. This equation was fitted to the experimental data using a non-linear least squares method in Matlab. In order to obtain a good fit quality for the important initial time steps, additional weight was given here (see results).

2.3.2. Model with a continuous relaxation spectrum

In the previous paper, the complex modulus was also fitted with a model with a continuous relaxation spectrum (see Section 2.4.2 in Aernouts and Dirckx, in press). This model is particularly well suited for describing materials with a rate-insensitive hysteresis loop. In this work, this model was also used to fit the reduced relaxation functions.

Consider again the standard linear solid, with stress relaxation function given by Eq. (1). For a continuous relaxation model, τ_1 is replaced by a continuous variable τ, the stiffness R_1 must be replaced by $R(\tau)\,d\tau$ and the equation must be integrated with respect to τ. Neubert (1963) proposed $R(\tau) = R/\tau$ for $\tau_1 \le \tau \le \tau_2$ (R being a constant) and $R(\tau)=0$ elsewhere so that

$$\begin{aligned}\sigma(t) &= R_0\varepsilon_0 + \varepsilon_0\left[\int_{\tau_1}^{\tau_2} \frac{R}{\tau} e^{-t/\tau}\, d\tau\right],\\ &= R_0\varepsilon_0 + R\varepsilon_0[E_1(t/\tau_2) - E_1(t/\tau_1)]\end{aligned} \tag{4}$$

with $E_1(z) = \int_z^\infty (e^{-t}/\tau)\, d\tau$ the exponential integral. Dividing by $\sigma_0 = R_0\varepsilon_0 + R\varepsilon_0 \ln(\tau_2/\tau_1)$ yields the reduced relaxation function for the continuous relaxation model

$$G(t) = g_\infty + c[E_1(t/\tau_2) - E_1(t/\tau_1)] \tag{5}$$

with constants g_∞, c, τ_1 and τ_2 and for which holds $G(0)=1$. This equation was fitted to the experimental data in the same way as described above. The exponential integral E_1 was numerically evaluated in Matlab using *expint*.

2.4. Viscoelastic finite element analysis

In the previous paper, we simulated the indentation experiments with specimen-specific finite element models (see Fig. 7 in Aernouts and Dirckx, in press). In these models, a linear elastic material was used and the stiffness increase as a function of indentation frequency was investigated.

In this work, the material properties in these finite element models were updated to model viscoelastic behavior. The simulations were performed with the finite element package FEBio. In FEBio, viscoelasticity is implemented based on the quasi-linear viscoelastic theory by Fung (1981) and Puso and Weiss (1998). This theory assumes that the viscoelastic stress can be written as a convolution of a relaxation function $G(t)$ with a purely elastic response σ^e:

$$\sigma(t) = \int_{-\infty}^{t} G(t-s)\frac{d\sigma^e}{ds}\, ds. \tag{6}$$