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The mathematical description of the body centre of mass 3D path in human and animal locomotion

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ABSTRACT

Although the 3D trajectory of the body centre of mass during ambulation constitutes the 'locomotor signature' at different gaits and speeds for humans and other legged species, no quantitative method for its description has been proposed in the literature so far. By combining the mathematical discoveries of Jean Baptiste Joseph Fourier (1768–1830, analysis of periodic events) and of Jules Antoine Lissajous (1822–1880, parametric equation for closed loops) we designed a method simultaneously capturing the spatial and dynamical features of that 3D trajectory. The motion analysis of walking and running humans, and the re-processing of previously published data on trotting and galloping horses, as moving on a treadmill, allowed to obtain closed loops for the body centre of mass showing general and individual locomotor characteristics. The mechanical dynamics due to the different energy exchange, the asymmetry along each 3D axis, and the sagittal and lateral energy recovery, among other parameters, were evaluated for each gait according to the present methodology. The proposed mathematical description of the 3D trajectory of the body centre of mass could be used to better understand the physiology and biomechanics of normal locomotion, from monopods to octopods, and to evaluate individual deviations with respect to average values as resulting from gait pathologies and the restoration of a normal pattern after pharmacological, physiotherapeutic and surgical treatments.

1. Introduction

All people walk in a similar way (this applies also to running or skipping) but we are able to promptly detect slight deviations from the common movement pattern and sometimes recognise through this someone we know. In particular, what we mainly spot are changes in the motion of the head/trunk segment, which reflect changes in the 3D path of the body centre of mass (BCOM). Apart from anecdotal and subjective evaluation of gait, there are a few professional categories specifically interested in the quantitative assessment of locomotion:

(1) Biomechanists have classically been interested in general features of gaits, such as the mechanical work necessary to sustain locomotion and the energy saving strategies to contain the metabolic cost of transport (e.g. Cavagna and Margaria, 1966; Alexander, 1989). Both aspects have been found to mostly relate to the 3D trajectory of BCOM, whose time-course (periodic raises and accelerations) affects the so-called 'external' mechanical work.
(2) Locomotion pathologists have commonly used inverse dynamics to estimate joint moments and powers, and compare them to standard values. It is intuitive, though, that the different types of *claudicatio* could be investigated, at the first glance, as deviations of BCOM trajectory from the 'normal' 3D path, and the success of a rehabilitative programme should be ultimately evaluated on the basis of the restoration of a healthy, almost symmetrical BCOM path.
(3) Comparative physiologists and theoretical biologists can use BCOM path description to infer the influence of basic bipedal locomotor paradigms (walking, hopping, and skipping) in animal evolution when the number of legs varies (from monopods to octopods).
(4) Biomedical and automation engineers get insights about the efficacy of locomotion, for bipedal/quadrupedal robots, from a quantitative analysis of the different gaits. The results obtained through a biomimetic design of legged machines can be tested by comparing their motion to the one of true biological 'vehicles'.

Thus there are many reasons to develop a clear methodological framework devoted to mathematically describe the experimentally captured motion of BCOM in 3D. A purposely designed set of equations would summarise both the general aspects of the gaits

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and the individual characteristics of movement, a sort of 'locomotor signature' capable to reflect any significant change in the motion pattern.

While BCOM is a very simple concept (i.e. the mass-weighted average of the position of all the body segments), its position is not fixed and can only be computed either by force platforms or by motion analysis systems. Although force platforms could provide BCOM trajectory irrespective of the anthropometric characteristics of the body, and the process is methodologically safer (forward dynamics involve integration, with benefits deriving from the associated signal smoothing), this methodology remains impractical due to: (1) the huge instrumented space necessary to capture only a few strides, and (2) the inherent speed variability. The combination of a (powerful and reliable) treadmill, which allows reproduction of walkway conditions at controlled speeds, and of a 4+ camera motion analysis system enables capture of a large sample of consecutive strides, from which the 'representative' stride can be obtained. Another advantage of such an experimental setup is the representation of BCOM trajectory as relative to its average forward translation, resulting in a compact 3D loop, where locomotion characteristics (speed changes, asymmetries, etc—see below) are more easily identifiable than in the 'absolute' trajectory. Also, differently from describing the gait as 3 apparently independent time courses of the BCOM trajectory coordinates, the closed 3D loop contains information about the functional interaction among them. The obtained BCOM loop can be compared with the average 'standard' ones to extract quantitative information about the effects of speed, gait, overall mass, gender, age, musculo-skeletal or neurologic injury/disease, and about the success of a rehabilitative, pharmacological or surgical procedure.

This paper is about an experimental and mathematical technique to reduce the complexity of BCOM 3D trajectory during locomotion to a manageable entity, whose quantitative description is expected to help in the fields of biomechanics, physiology and pathology of gait. Some examples of potential applications will also be provided.

2. Methods

2.1. Ethical approval

The local ethics committee of the University of Milan approved the project after checking that the study procedures conformed to the standards set by the latest revision of the Declaration of Helsinki.

2.2. Experimental procedures

As documented in many previous papers on humans and horses (e.g. Minetti et al., 1993, 1999, or see Saibene and Minetti, 2003 for a review of them), the experimental BCOM trajectory is obtained by combining the 3D coordinates of 12 body segments, measured at 100 Hz on a treadmill by a motion analyser (ELITE System, BTS, Milan, Italy), with the anthropometric/equimetric tables (e.g. Dempster, 1955; Dempster et al., 1959; Buchner et al., 1997) reporting the fractional mass of the different segments and its relative position within them. Most of the data processing described in this paper has been obtained by custom programs written in LabView (versions 2.0–7.1, National Instruments, Austin, US).

Eleven male subjects (age 27.7 ± 4.6 yrs, mass 80.4 ± 10.2, and stature 181.6 ± 6.6 cm), who signed the informed consent to participate the study, were analysed during walking at 5 speeds (3–7 km/h, step 1 km/h) and running at 9 speeds (7–15 km/h, step 1 km/h). At each speed body segments kinematics was sampled for 20–30 s, so as to measure a consistent number of consecutive strides.

A subset of each kinematic sample, containing an integer number of strides, was obtained by inspecting the periodicity of vertical coordinates of body segments. Individual strides were then extracted by searching the maxima in the vertical coordinates of BCOM and choosing the begin/end frame in the time corresponding to every other value of the maxima (due to the double periodicity within a stride). A total of 1120 strides were analysed.

2.3. Mathematical processing

By having sampled the body motion on a treadmill, the trajectory of the BCOM during each stride is expected to follow a Lissajous contour, i.e. a convoluted loop showing its 3D displacement with respect to the average position. The advantages of a parametric representation (as the Lissajous contour) of the BCOM trajectory are that: (a) the fourth variable, namely the time, is retained and allows the 3D visualisation of the movement dynamics, (b) the differentiation/integration of the trajectory can be inferred in order to obtain speeds, energies and path lengths, and (c) whichever regression model is chosen to describe the time courses of the x (progression axis), y (vertical axis) and z (lateral axis) coordinates, the accuracy of the 3D fit benefits from the simultaneous equations, with the need of only a few regression coefficients per coordinate regardless of the complexity of the path.

The time course of each of the 3 BCOM coordinates was fit by a Fourier Series, truncated at the 6th harmonic (see below), with the time as the independent variable. The advantage of a truncated Fourier Analysis, apart from the periodical nature of these equations, is that, differently from a polynomial regression, it is insensitive to further refinements, i.e. the sine coefficients published in this paper will still be valid even when, in future studies, a greater number of harmonics would be introduced.

Following equations describes the detailed mathematical processing of the experimental 3D trajectory of BCOM, while further details and the List of Abbreviations are provided in the Electronic Supplementary Material as an Appendix A of this paper.

Each extracted stride, with period T, has been forced to become close loops, i.e.

$$x(T)=x(0),\quad y(T)=y(0),\quad z(T)=z(0)$$

by imposing the transformation:

$$x(t')=x(t')-\frac{t'}{T}\Delta_x,\quad y(t')=y(t')-\frac{t'}{T}\Delta_y,\quad \text{and } z(t')=z(t')-\frac{t'}{T}\Delta_z$$

where

$$\Delta_x=x(T)-x(0),\quad \Delta_y=y(T)-y(0),\quad \text{and } \Delta_z=z(T)-z(0)$$

and t' is the absolute chronological time (s).

The x, y and z coordinates of each stride undertook a Fourier Analysis truncated to the 6th harmonic, resulting in:

$$\hat{x}(t)=a_0^x+\sum_{i=1}^{6}a_i^x\sin(it)+b_i^x\cos(it),$$

$$\hat{y}(t)=a_0^y+\sum_{i=1}^{6}a_i^y\sin(it)+b_i^y\cos(it)$$

and

$$\hat{z}(t)=a_0^z+\sum_{i=1}^{6}a_i^z\sin(it)+b_i^z\cos(it)$$

where

$$t=2\pi\frac{t'}{T}$$

and i is the harmonic number. The average of the vertical coordinate, i.e. a_0^y, for each stride is collected as to calculate the mean height value (for n strides) at each speed as:

$$\bar{a}_0^y=\frac{\sum_{j=1}^{n}a_{0,j}^y}{n}$$

while a_0 constants for x and z coordinates have been removed from the analysis (this corresponds to consider the progression and the lateral data of the stride as centred about the origin of those axes).

Since

$$a\sin(t)+b\cos(t)=c\sin(t+\phi)$$

where

$$c=\sqrt{a^2+b^2}\quad \text{and}\quad \phi=\frac{\pi}{2}\operatorname{sgn}(b)-\arctan\left(\frac{b}{a}\right)$$

the Fourier Series can be also expressed, after temporarely removing the a_0 constant for the vertical axis, as:

$$\hat{x}(t)=\sum_{i=1}^{6}c_i^x\sin(it+\phi_i^x),\quad \hat{y}(t)=\sum_{i=1}^{6}c_i^y\sin(it+\phi_i^y)\quad \text{and}$$

$$\hat{z}(t)=\sum_{i=1}^{6}c_i^z\sin(it+\phi_i^z)$$

This is called the 'phase angle form' of the Fourier Series and is more convenient than the original form as it contains just sine functions. The motion of BCOM would exhibit perfect right–left symmetry if it contained just even harmonics in the x (progression) and y (vertical) directions, and just odd