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Peristaltic transport in a channel with a porous peripheral layer: model of a flow in gastrointestinal tract

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Abstract

Peristaltic transport in a two dimensional channel, filled with a porous medium in the peripheral region and a Newtonian fluid in the core region, is studied under the assumptions of long wavelength and low Reynolds number. The fluid flow is investigated in the waveframe of reference moving with the velocity of the peristaltic wave. Brinkman extended Darcy equation is utilized to model the flow in the porous layer. The interface is determined as a part of the solution using the conservation of mass in both the porous and fluid regions independently. A shear-stress jump boundary condition is used at the interface. The physical quantities of importance in peristaltic transport like pumping, trapping, reflux and axial velocity are discussed for various parameters of interest governing the flow like Darcy number, porosity, permeability, effective viscosity etc. It is observed that the peristalsis works as a pump against greater pressure in two-layered model with a porous medium compared with a viscous fluid in the peripheral layer. Increasing Darcy number *Da* decreases the pumping and increasing shear stress jump constant β results in increasing the pumping. The limits on the time averaged flux $\bar{Q}$ for trapping in the core layer are obtained. The discussion on pumping, trapping and reflux may be helpful in understanding some of the fluid dynamic aspects of the transport of chyme in gastrointestinal tract.

Keywords: Peristaltic pumping; Trapping; Reflux; Porous medium; Shear stress jump condition

1. Introduction

Peristaltic transport is a form of fluid transport generated by a progressive wave of area contraction or expansion along the length of a distensible tube containing fluid. Peristalsis induces in general propulsive and mixing movements. The mechanism is found in many biological systems having smooth muscle tubes for example, the movement of chyme in the gastro-intestinal tract, intra-uterine fluid motion, vasomotion of small blood vessels and the flows in many other glandular ducts. The developments on mathematical modelling and experimental fluid mechanics of peristaltic flows was given in an early excellent review by Jaffrin and Shapiro (1971).

It is observed in many biological ducts undergoing peristalsis, the inner walls of the boundary are coated with a fluid having different properties from that of the pumped core fluid, Keener and Sneyd (1998). The primary function of the gastrointestinal tract is to absorb nutrients from the mix of food and liquid that move through it. It is surrounded by a number of muscle layers having smooth muscles and contraction of these muscle layers can mix the contents of the tract and move food in a controlled manner in an appropriate direction. Beneath the muscle layers is the submucosa and finally a layer of epithelial cells, which are the responsible for the absorption of water from the intestine. It consists of many folds and there are pores through the tight junctions of them. Motivated by these facts we model the flow in gastrointestinal tract qualitatively by a peristaltic flow of two fluid systems

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Nomenclature

X, Y	Cartesian coordinates in the laboratory frame
x, y	Cartesian coordinates in the wave frame
a	half mean width of the channel
b	amplitude of the peristaltic wave
c	speed of the peristaltic wave
$H(X-ct)$	travelling wave form in the laboratory frame
$H_1(X-ct)$	interface wave form in between the core and peripheral layer
U_i, V_i	velocity components in the laboratory frame, $i=1$ denotes the core and $i=2$ denotes the peripheral layer
u_i, v_i	velocities components in the wave frame
p_i, P_i	pressure in the core and peripheral layer
ψ_i	stream function in the wave frame
μ_1, μ_2	core and peripheral layer viscosities
ρ_1, ρ_2	core and peripheral layer densities
ε	porosity
k	permeability
Re, Da	Reynolds number and Darcy number
β	shear-stress jump constant
ϕ	ratio of amplitude of the wave to the half mean width of the channel
ρ	ratio of density of peripheral layer to the core layer
μ	the ratio of viscosity of peripheral layer to the core layer
h, h_1	non-dimensional peristaltic wave form and the interface wave form
δ	ratio of half mean width of the channel to the wavelength
ΔP	dimensionless pressure drop across the wavelength
$\bar{Q}$	dimensionless time averaged flux in the laboratory frame
γ	the initial value of the interface

in a channel with a porous peripheral layer and a Newtonian fluid core layer.

The boundary conditions to be satisfied at the interface of a two fluid system are the matching of tangential velocity, normal velocity, shear stress and normal stress. Beavers and Joseph (1967) have investigated the fluid flow at the interface between a porous medium and fluid layer in an experimental study and proposed a slip in velocity at the interface. There exist numerous subsequent studies in the literature which suggest different boundary conditions at the interface between porous and fluid layers (Chen and Chen, 1992; Neale and Nader, 1974; Poulikakos and Kazmierczak, 1987; Saffman, 1971; Vafai and Kim, 1990). Ochoa-Tapia and Whitaker (1995) introduced a boundary condition which accounts for the jump in the shear stress at the interface between a porous and fluid layer by applying a sophisticated volume averaging technique and this resolves the problem of over determining the physical problem discussed by Nield (1991) and Vafai and Kim, (1995). Kuznetsov (1996,1997) utilized the significance of the shear stress jump condition at the interface to discuss the fluid flow in a channel partially filled with a porous medium. Recently Alazmi and Vafai (2001) investigated the fluid flow and heat transfer between a porous medium and a fluid layer by considering various types of interfacial matching of shear stress conditions proposed in the literature.

The mathematical modeling of the two-fluid system involves the determination of the interface between the core and peripheral layers. Peristaltic transport in two immiscible layers of fluid has been investigated for a channel by Brasseur et al. (1987), and for a circular tube by Ramachandra Rao and Usha (1995). They determined the interface by considering mass conservation in both core and peripheral layers independently and a similar analysis is followed in this paper.

In the present investigation, we study the peristaltic transport of two-layered system with a porous peripheral layer and a core viscous fluid. The Brinkman extended Darcy equations have been considered for the porous medium and the shear stress jump boundary condition of Ochoa-Tapia and Whitaker (1995) is used at the interface between porous and fluid regions together with continuity of velocity and normal stress conditions. The interface is determined by solving a transcendental equation, derived through the conservation of mass in both core and peripheral region, using Matlab packages. The trapping and pumping characteristics are discussed for different new parameters, such as Darcy number Da, porosity ε, and shear stress jump β arising due to a porous peripheral layer. In the limit $Da \to \infty$, we recover the results of Brasseur et al. (1987) for two viscous fluid layers.

2. Mathematical formulation

Consider the peristaltic transport in a two dimensional channel, with a porous medium in the peripheral layer and an incompressible Newtonian fluid in the core region, Fig. 1. We assume the porous medium is isotropic and homogeneous. The channel wall is flexible and an infinite wave train is moving on the walls of amplitude b and wavelength λ in the axial direction with a constant speed c. The walls are taken by $Y = \pm H(X-ct)$ in Cartesian coordinate system (X, Y) with t as the time. The mean width of the channel is $2a$ and