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Exploiting parameter sparsity in model-based reconstruction to accelerate proton density and T_2 mapping

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ABSTRACT

T_2 mapping is a powerful noninvasive technique providing quantitative biological information of the inherent tissue properties. However, its clinical usage is limited due to the relative long scanning time. This paper proposed a novel model-based method to address this problem. Typically, we directly estimated the relaxation values from undersampled k-space data by exploiting the sparse property of proton density and T_2 map in a penalized maximum likelihood formulation. An alternating minimization approach was presented to estimate the relaxation maps separately. Both numerical phantom and in vivo experiment dataset were used to demonstrate the performance of the proposed method. We showed that the proposed method outperformed the state-of-the-art techniques in terms of detail preservation and artifact suppression with various reduction factors and in both moderate and heavy noise circumstances. The superior reconstruction performance validated its promising potential in fast T_2 mapping applications.

1. Introduction

MR relaxometry (e.g., T_1/T_2 mapping) provides a noninvasive quantitative manner to access tissue structure and composition, water content and iron levels. It is extensively used in research studies of iron overload [1], cartilage disease [2], multiple sclerosis [3], myocardial infarction [4], cancer [5,6], etc. However, one of the major difficulties of its yet not being widely applied in clinic is the relative long scanning time since usually multiple images need to be acquired sequentially. Take the T_2-weighted image series for an example, the signal acquisition scheme can be mathematically expressed as:

$$d_l(k) = \int \rho_l(x) \exp(-i2\pi k \cdot x)\, dx + n_l(k)$$

where $\rho_l(x)$ is the desired image function at the l-th echo time. $d_l(k)$ is the measured k-space data and $n_l(k)$ denotes the complex Gaussian noise. Conventional imaging methods require a fully sampled k-space for all l.

Fast imaging techniques grounded on the theory of sparse sampling have shown promising potential in accelerating MR acquisitions. Prior information, mostly the image sparsity [7] and spatiotemporal partial separability [8], has been exploited to constrain the solution space of the desired image function from the undersampled data. Based on similar assumptions (i.e., sparsity and partial separability), a number of sparse reconstruction methods [9–15] have been developed with various variations regarding the image model, sparsifying transform, regularization, etc. Specifically, authors in literature [9] proposed to learn an overcomplete dictionary to sparsify the signal. The approach was verified in T_1 and T_2 mapping in the brain with highly reduced data. The study in literature [10] used the smoothness of signal evolution in the parametric dimension to accelerate variable flip angle T_1 mapping. Similar idea was developed in literature [13] to enable fast T_1 mapping of the mouse heart. In literature [11,12], principle component decomposition played as the sparsifying transform along the parametric direction, while it was used to linearize the signal model in literature [15]. In literature [14], the authors proposed to model the entire image series as a partial separable function, assuming that the spatial-parametric image matrix has a low rank. All the above methods require a parameter fitting step afterwards

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to extract the relaxation values (i.e., T_1, T_2) based on the intrinsic parametric model. Generally, the relaxation values are obtained by performing pixel-wise single exponential curve least square fitting. More accurate estimation can be achieved by using advanced technique considering the Rician distribution of the magnitude data [16]. However, since the procedure of sparse reconstruction is nonlinear, unpredictable errors would occur with large reduction factors and noisy measurements. These errors could probably further propagate into the subsequent parameter estimation.

Another type of approach for parameter mapping from undersampled measurements is the model-based (MB) reconstruction method [17–21], which directly estimates the relaxation values from the undersampled *k*-space data. The superior performance of the MB method owes to the fact that the number of unknowns in the relaxation map is much less than the total number of image pixels in the image series. Additionally, prior knowledge can also be employed to reduce artifacts and further improve the reconstruction quality. For instance, the work in reference [18] used total variation to promote image sparsity and authors in reference [19] penalized the L_2 norm of the finite differences of the relaxations' Fourier transform to enforce smoothness of the relaxation map.

In this work, we proposed an alternating minimization method for model-based proton density and T_2 mapping with parameter sparsity constraint (AM-MBPS). Typically, parameter sparsity, modeled as the L_1 norm of corresponding sparse coefficients, was penalized to promote the sparsity of proton density and T_2 map simultaneously. Each relaxation map was estimated separately in an alternating minimization fashion (i.e., keep one fixed while solving for the other). Thus, on top of the MB method, the proposed MBPS method may significantly reduce the required number of measurements and improve reconstruction quality.

A similar approach was taken and reported recently, and independently, by Zhao et al. [21]. Basically, there are three differences distinguishing the two works. First, according to the formulation in [21], only the sparsity constraint of the T_2 map was penalized, which is relatively easier to implement since only one regularization parameter need to be tuned. But regularizations on both proton density and T_2 map as in our work will result in more improved reconstruction. Secondly, the proton density and T_2 map were estimated jointly in [21] while we propose to solve them separately. Joint estimation may cause a poorly scaled problem as described in the discussion. Thirdly, we formulated the reconstruction as an unconstraint L_1 norm minimization problem while the authors in [21] formulated it as a L_0 quasi-norm constrained optimization problem. Though efficient greedy algorithms can be used to solve the L_0 quasi-norm problem, the exact sparsity level of the relaxation map may not be known as a prior.

2. Materials and methods

2.1. Model-based formulation

With proper discretization, the image acquisition scheme can be expressed in matrix-vector form as:

$$\mathbf{d}_l = \mathbf{F_u}\boldsymbol{\rho}_l + \mathbf{n}_l \tag{1}$$

where $\mathbf{d}_l \in \mathbb{C}^{M\times 1}$ and $\boldsymbol{\rho}_l \in \mathbb{C}^{N\times 1}$ are the undersampled measurements and desired T_2-weighted image at the l-th echo time respectively. $l = 1, 2, \ldots, L$, L is the echo train length and N is the total number of image pixels. $\mathbf{F_u} \in \mathbb{C}^{M\times N}$ denotes the undersampled Fourier encoding matrix with $M \ll N$. $\mathbf{n}_l$ is the observation noise. Given the assumption that $\mathbf{n}_l$ is complex white Gaussian, the maximum likelihood solution of $\boldsymbol{\rho}_l$ can be obtained by solving the simple least-squares problem:

$$\left(\hat{\boldsymbol{\rho}}_1, \hat{\boldsymbol{\rho}}_2, \ldots, \hat{\boldsymbol{\rho}}_L\right) = \underset{\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\ldots,\boldsymbol{\rho}_L}{arg\min} \sum_{l=1}^{L} \left\|\mathbf{F}_u\boldsymbol{\rho}_l - \mathbf{d}_l\right\|_2^2 \tag{2}$$

In T_2 mapping, the image function $\boldsymbol{\rho}_l$ generated using a standard Carr-Purcell-Meiboom-Gill (CPMG) spin echo can be written as:

$$\boldsymbol{\rho}_l = \boldsymbol{\rho}_{PD} \cdot \exp(-l\Delta t\boldsymbol{\beta} + i\boldsymbol{\varphi}) \tag{3}$$

where $\boldsymbol{\rho}_{PD} \in \mathbb{R}^{+N\times 1}$ represents the proton density distribution function, Δt is the echo time spacing, $\boldsymbol{\varphi}$ is the image phase shared by the image series. $\boldsymbol{\beta} \in \mathbb{R}^{+N\times 1}$ denotes the R_2 map. Operator $\cdot$ stands for element-wise multiplication. Substituting Eq. (3) into Problem (2), the maximum likelihood estimates of the relaxations can be obtained as:

$$(\hat{\boldsymbol{\rho}}_0, \hat{\boldsymbol{\beta}}) = \underset{(\boldsymbol{\rho}_0,\boldsymbol{\beta})}{arg\min} \sum_{l=1}^{L} \left\|\mathbf{F}_u\boldsymbol{\rho}_0 \cdot \exp(-l\Delta t\boldsymbol{\beta}) - \mathbf{d}_l\right\|_2^2 \tag{4}$$

where $\boldsymbol{\rho}_0 = \boldsymbol{\rho}_{PD} \cdot \exp(i\boldsymbol{\varphi})$. The proton density and T_2 map can be obtained afterwards via $\boldsymbol{\rho}_{PD} = \left|\boldsymbol{\rho}_0\right|$ and $T_2 = 1./\boldsymbol{\beta}$.

2.2. Proposed method

In this work, we assumed that the relaxation maps could be sparser than conventional MR images. This is probably because the content of conventional MR images is affected by several factors, including the intrinsic contrast mechanism (i.e., proton density T_1, T_2, $T_2{}^*$ weighting) and the hardware conditions (i.e., coil sensitivity, B_0 inhomogeneity). While the quantitative relaxation map accessing each contrast component is solely tissue property dependent. The relaxation map should be sparser since the signal variation in each contrast component is much less than that in conventional MR images.

Thus, we propose to incorporate the sparsity constraint of proton density and T_2/R_2 map into the MB formulation, yielding a penalized maximum likelihood solution:

$$(\hat{\boldsymbol{\rho}}_0, \hat{\boldsymbol{\beta}}) = \underset{(\boldsymbol{\rho}_0,\boldsymbol{\beta})}{arg\min} \sum_{l=1}^{L} \left\|\mathbf{F_u}\boldsymbol{\rho}_0 \cdot \exp(-l\Delta t\boldsymbol{\beta}) - \mathbf{d}_l\right\|_2^2 + \lambda_1\left\|\boldsymbol{\Psi}_w\boldsymbol{\rho}_0\right\|_1 + \lambda_2\left\|\mathbf{D}\boldsymbol{\beta}\right\|_1 \tag{5}$$

L_1-regularization was employed to enforce sparsity. Appropriate sparsifying transforms were selected for proton density and the R_2 map. Typically, $\boldsymbol{\Psi}_w \in \mathbb{C}^{N\times N}$ is the wavelet transform (Daubechies 4) and $\mathbf{D} = \left[\mathbf{D}_x, \mathbf{D}_y\right]$ with $\mathbf{D}_x$ and $\mathbf{D}_y$ denoting the forward finite difference operators on the first and second coordinates respectively. Other sparsifying transforms are also feasible. λ_1 and λ_2 are regularization parameters controlling the trade-off between data consistency and sparsity constraint.

To solve problem (5), we employed an alternating minimization approach. Specifically, the solution of problem (5) was found by iteratively solving the following two sub-problems:

$$\hat{\boldsymbol{\rho}}_0^{(k)} = \underset{\boldsymbol{\rho}_0}{arg\min} \sum_{l=1}^{L} \left\|\mathbf{F_u}\boldsymbol{\rho}_0 \cdot \exp(-l\Delta t\boldsymbol{\beta}^{(k)}) - \mathbf{d}_l\right\|_2^2 + \lambda_1\left\|\boldsymbol{\Psi}_w\boldsymbol{\rho}_0\right\|_1 \tag{6}$$

$$\hat{\boldsymbol{\beta}}^{(k+1)} = \underset{\boldsymbol{\beta}}{arg\min} \sum_{l=1}^{L} \left\|\mathbf{F_u}\boldsymbol{\rho}_0{}^{(k)} \cdot \exp(-l\Delta t\boldsymbol{\beta}) - \mathbf{d}_l\right\|_2^2 + \lambda_2\left\|\mathbf{D}\boldsymbol{\beta}\right\|_1 \tag{7}$$