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Structural analysis of optimal investment for firms with non-concave revenue

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Abstract

Qualitative properties of optimal investment strategies for a firm with quadratic costs that faces network externalities in its revenues are analysed. Organising the qualitative information in a bifurcation diagram, it is found that the structure of the bifurcation manifolds is determined by a so-called swallow-tail singularity. This implies the existence of up to three saddle equilibria of the state-costate system, whose local optimal stability for positive discount rates is determined by a three-dimensional bifurcation diagram. Taking a particular two-dimensional section of this diagram, it is shown that the bifurcation curves divide this into 19 different parameter regions; within each region, the corresponding phase systems, and also the optimal solutions derived from them, are structurally stable. In particular, stable configurations with threshold (Skiba) points are found in this way.

JEL classification: C61; D92

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1. Introduction

If at a given point in time, a firm's capital stock is not optimal (that is, if the marginal product of capital does not equal the marginal replacement cost of capital) a manager must decide about investment or disinvestment. Under the assumption that there are adjustment costs to be paid on changing the capital stock, it is not optimal to try to jump to the optimal level all at once; rather, the manager has to make a long-term investment plan. That firms

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do not change their capital stock instantaneously has already been claimed by Keynes (1936); the work of Eisner and Strotz (1963), Lucas (1967), Gould (1968) and Treadway (1969) has shown how “Keynesian” investment schedules can be derived from optimality considerations by assuming adjustment costs of capital.

Tobin (1969) and Tobin and Brainard (1977) have formulated these investment schedules in terms of what is generally known as “Tobin’s *q*,” which denotes the quotient of marginal product by marginal replacement cost of capital; Tobin and Brainard (p. 238) state that “Values of *q* above 1 should stimulate investment (. . .) and values below 1 discourage investment”. This can easily be translated into an adaptive investment strategy, which is plausible and sufficient if there is a single optimal long-term steady state of the investment problem.

Increasing returns to scale over some regions of the firm size complicate the picture. In the model treated below, they are caused by network externalities, but there may be other reasons for them. In the presence of increasing returns, it may happen that it is optimal to invest even if the value of *q* is below 1. However, for vanishing, and by approximation for very small discount rates, there is still a relation between the optimal investment plan and Tobin’s *q*: on an optimal growth path, investment has to be such that the growth of capital accelerates if *q* is below 1 and decelerates if *q* is above 1.

More generally, for positive discount rates increasing returns may lead to the existence of several optimal long-term steady states, the initial size of the firm determining to which of these states an optimal investment plan will lead. This divides the possible initial sizes into ‘optimal basins of attraction’ of the respective long-term steady states; these basins are separated by threshold points, which are often called Skiba or Dechert-Nishimura-Skiba points, after the seminal articles of Skiba (1978) and Dechert and Nishimura (1983). For a good overview of the literature on threshold points, see Deissenberg et al. (2004).

Whether such thresholds exist for a given situation is a delicate non-linear problem, which depends on the convexity of the cost function and the non-concavity of the production function. In the following pages the problem is investigated by parametrising the strength of the non-concavity and the shape of the revenue function to obtain a detailed picture of the relative effects of these properties. This picture takes the mathematical form of a multi-dimensional bifurcation diagram.

Previously (Wagener, 2003) it has been shown that knowledge of the bifurcations of the state-costate system associated with the optimality problem contains enough information to determine whether or not, for given values of the parameters, there are threshold points in the system, provided the state variable of the problem is one-dimensional; the set of parameters for which threshold points exist was shown to be bounded by manifolds of saddle-node and heteroclinic bifurcations. This relation is general, holding for all dimensions of the state space (Wagener, in preparation).

A major problem in the investigation of non-linear systems is that there are usually no global results to be had unless there is some additional structure present. In the present case, for small discounting, the state-costate equations are close to a so-called Hamiltonian system; this Hamiltonian structure places strong restrictions on the phase flow, and methods from perturbation theory can be applied to the problem.

Though the main thrust of the following will be the analysis of a well-chosen model system, it should be noted that the results obtained are robust, structurally stable in the mathematical sense.