Journal of Experimental Child Psychology 116 (2013) 829–838

Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

Numerical approximation abilities correlate with and predict informal but not formal mathematics abilities

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ARTICLE INFO

Article history:
Received 19 November 2012
Revised 25 July 2013
Available online 25 September 2013

Keywords:
Approximate number system (ANS)
Informal math
Formal math
Individual differences
Nonsymbolic numerical comparison
Longitudinal study

ABSTRACT

Previous research has found a relationship between individual differences in children's precision when nonverbally approximating quantities and their school mathematics performance. School mathematics performance emerges from both informal (e.g., counting) and formal (e.g., knowledge of mathematics facts) abilities. It remains unknown whether approximation precision relates to both of these types of mathematics abilities. In the current study, we assessed the precision of numerical approximation in 85 3- to 7-year-old children four times over a span of 2 years. In addition, at the final time point, we tested children's informal and formal mathematics abilities using the Test of Early Mathematics Ability (TEMA-3). We found that children's numerical approximation precision correlated with and predicted their informal, but not formal, mathematics abilities when controlling for age and IQ. These results add to our growing understanding of the relationship between an unlearned nonsymbolic system of quantity representation and the system of mathematics reasoning that children come to master through instruction.

Introduction

To succeed in school mathematics, children need to master a variety of skills. These skills include informal mathematics abilities such as numbering and counting, comparing numbers to determine which is more or less, and calculating the answers to simple arithmetic problems using tokens or

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0022-0965/$ - see front matter
http://dx.doi.org/10.1016/j.jecp.2013.08.003

fingers (Ginsburg & Baroody, 2003; Jordan, Kaplan, Ramineni, & Locuniak, 2009; National Mathematics Advisory Panel, 2008). These skills also include formal school-taught abilities that require adherence to the formal conventions of mathematics and include, for example, the ability to read and write Arabic numerals, an understanding of the place value system, and the ability to recall memorized addition, subtraction, and multiplication facts (Ginsburg & Baroody, 2003; Jordan et al., 2009; National Mathematics Advisory Panel, 2008).

In addition to these informal and formal mathematics skills, all of which typically require thinking about or manipulating number symbols (e.g., number words or digits), children also have access to a nonsymbolic prelinguistic system of numerical representation. This system can be used to approximate numerical quantities, compare approximate numerical representations, and perform approximate arithmetic operations such as addition and subtraction (Barth, La Mont, Lipton, & Spelke, 2005; Dehaene, 1992; Feigenson, Dehaene, & Spelke, 2004). This approximate number system (ANS) is present in humans from birth onward (Halberda, Ly, Willmer, Naiman, & Germine, 2012; Izard, Sann, Spelke, & Streri, 2009) and has been demonstrated in a variety of nonhuman animals (Brannon & Merritt, 2011). A hallmark feature of the ANS is that the imprecision in its representations increases as numerosity grows. As a consequence, the discriminability between two approximate number representations is determined by the ratio between them, not by their absolute difference (such performance is also commonly described as adhering to Weber's law). Although ANS representations remain noisy and imprecise throughout the lifespan (Halberda et al., 2012), numerous studies have found that the precision of ANS representations increases with age (Halberda & Feigenson, 2008; Halberda et al., 2012; Libertus & Brannon, 2010; Xu & Spelke, 2000). Even so, there are large differences in ANS precision between individuals of similar age. These individual differences are already present and stable during infancy (Libertus & Brannon, 2010; Libertus, Brannon, & Woldorff, 2011) and can be found across the entire lifespan (Halberda et al., 2012).

Previous research has revealed a small but stable relationship between these individual differences in ANS precision and mathematics performance in both children and adults (for a review, see Feigenson, Libertus, & Halberda, 2013). For example, Halberda, Mazzocco, and Feigenson (2008) found that students' mathematics abilities from kindergarten through sixth grade (measured using standardized math assessments) correlated significantly with their ANS precision measured at 14 years of age. Importantly, this relationship remained robust even when controlling for other cognitive abilities such as general intelligence, visuo–spatial skills, and working memory, thereby suggesting a fairly specific relationship between the ANS and mathematics ability. Furthermore, recent studies showed that ANS precision and mathematics performance are already linked in preschool-aged children, before the onset of rigorous mathematics instruction (Libertus, Feigenson, & Halberda, 2011), and that ANS precision measured in preschool predicts later mathematics performance (Libertus, Feigenson, & Halberda, 2013; Mazzocco, Feigenson, & Halberda, 2011). Finally, this link appears to persist even into adulthood (Halberda et al., 2012; Libertus, Odic, & Halberda, 2012).

Although it is still unclear what mechanisms may support a link between ANS precision and mathematics abilities, several possibilities have been raised. One is that the link may reside in children's intuitive arithmetic operations (Gilmore, McCarthy, & Spelke, 2007, 2010). Another possibility is that it is created through a mapping between the ordinal relations of the ANS and ordinal relations among number symbols (Lyons & Beilock, 2011). Finally, it may arise during the acquisition of number symbol meanings and during online access of those meanings (De Smedt, Verschaffel, & Ghesquiere, 2009; Holloway & Ansari, 2009; Rousselle & Noel, 2007; Sasanguie, De Smedt, Defever, & Reynvoet, 2012).

An important step toward evaluating these possibilities is obtaining a clearer characterization of the specific mathematics abilities that are linked to ANS precision. In particular, the relationship between ANS precision and formal and informal mathematics abilities has yet to be elucidated. As described above, mathematics has often been conceived as including both formal and informal concepts and skills (e.g., Baroody, 1987; Raman, 2002), and this distinction between formal and informal mathematics abilities has played an important role in investigations of mathematics learning impairments (Mazzocco & Thompson, 2005; Russell & Ginsburg, 1984). This raises the question of whether ANS precision relates only to informal mathematics abilities, only to formal abilities, or to some combination of the two. Currently, no research has examined the extent to which formal versus informal mathematics abilities relate to the precision of the ANS.