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How do we convert a number into a finger trajectory?

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ABSTRACT

How do we understand two-digit numbers such as 42? Models of multi-digit number comprehension differ widely. Some postulate that the decades and units digits are processed separately and possibly serially. Others hypothesize a holistic process which maps the entire 2-digit string onto a magnitude, represented as a position on a number line. In educated adults, the number line is thought to be linear, but the "number sense" hypothesis proposes that a logarithmic scale underlies our intuitions of number size, and that this compressive representation may still be dormant in the adult brain. We investigated these issues by asking adults to point to the location of two-digit numbers on a number line while their finger location was continuously monitored. Finger trajectories revealed a linear scale, yet with a transient logarithmic effect suggesting the activation of a compressive and holistic quantity representation. Units and decades digits were processed in parallel, without any difference in left-to-right vs. right-to-left readers. The late part of the trajectory was influenced by spatial reference points placed at the left end, middle, and right end of the line. Altogether, finger trajectory analysis provides a precise cognitive decomposition of the sequence of stages used in converting a number to a quantity and then a position.

1. Introduction

The invention of multi-digit numbers is a major achievement in our culture. It took mankind centuries to develop the idea that large numbers can be represented with merely 10 symbols by relying on their relative positions. During education, the human brain learns the decimal system and, ultimately, it becomes very intuitive that the digit 4 in 41 stands for four decades, while the digit 4 in 14 stands for four units. But what is it exactly that we understand? How does our brain represent multi-digit quantities, and what are the processes that convert a sequence of digit symbols into this quantity representation? In spite of our growing knowledge of the cognitive and neurological brain mechanisms of numerical cognition, the issue of multi-digit quantities was addressed by relatively few studies, and even fewer have investigated the processes that convert digits into these quantities. The present research explored these issues, and was centered on three major questions: holistic vs. decomposed encoding of multi-digit quantities, the use of a logarithmic or a linear quantity scale, and sequential vs. parallel processing of the digits in multi-digit numbers. In investigating these questions we aimed not only to describe the various cognitive representations of numbers in educated adults, but also to dissect the successive stages by which multi-digit Arabic numbers are converted into quantities.

1.1. Holistic vs. decomposed quantity representation

One of the main disputes about two-digit quantity representation is between the holistic and decomposed

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approaches. The holistic approach claims that two-digit numbers are represented as holistic quantities: similarly to single digits, two-digit numerals are recognized as a whole and mapped onto a memorized quantity (Dehaene, Dupoux, & Mehler, 1990; Reynvoet & Brysbaert, 1999). The decomposed approach proposes that when a person deals with symbolic multi-digit numerals, only the quantities associated with the individual digits are activated and manipulated (Nuerk & Willmes, 2005). For example, the decomposed approach postulates that comparing two 2-digit numbers is achieved using two separate comparisons – one of the decade digits and another of the unit digits (Meyerhoff, Moeller, Debus, & Nuerk, 2012; Moeller, Fischer, Nuerk, & Willmes, 2009; Nuerk & Willmes, 2005).

The holistic-decomposed debate often made use of the fact that it takes longer to compare two digits when they are farther apart (Moyer & Landauer, 1967). This distance effect was taken to show that the comparison is performed by converting numbers from the decimal notation to an internal quantity code. The holistic model was supported by the finding of a continuous distance effect even in a comparison task where participants had to compare two-digit number targets to a fixed reference such as 55 (Brysbaert, 1995; Dehaene et al., 1990). Crucially, the unit distance affected the comparison time even when the decade digits were different (e.g., comparing 69 with 55 is faster than comparing 61 with 55), and in certain experimental settings there was no discontinuity at decade boundaries (Dehaene, 1989; Dehaene et al., 1990; Hinrichs, Yurko, & Hu, 1981). To account for this finding, a decomposed model must assume that the unit digits are compared even when they are numerically irrelevant, and that an incompatible unit comparison result interferes with the decade comparison and slows it down. Such an explanation predicts that if the onset of the unit digits is manipulated to be slightly earlier than the decade digit onset, the irrelevant unit comparison should have greater effect and therefore increase its interference in RT. This prediction was refuted, thereby supporting the holistic model (Dehaene et al., 1990).

In a slightly different comparison task, however, in which the subjects have to decide which of two simultaneously presented 2-digit numbers was the larger, the decomposed approach was supported by the discovery that the distance effect is modulated by decade-unit compatibility: for equal overall distance, pairs of two-digit numbers are compared faster when the units comparison result is compatible with the two-digit comparison result (e.g., 32 vs. 47, where 2 is smaller than 7) than when the units comparison is incompatible (e.g., 37 vs. 52, where 7 is larger than 2). The decomposed model can explain this compatibility effect as an interference from the incompatible unit comparison (Macizo, Herrera, Román, & Martín, 2011; Nuerk, Kaufmann, Zoppoth, & Willmes, 2004; Nuerk, Weger, & Willmes, 2001; Nuerk & Willmes, 2005). The holistic model cannot explain the compatibility effect because such a model considers only the overall distance between the compared numbers. The decomposed model was also supported by a recent study that showed a unit digit quantity effect in two-digit number bisection (Doricchi et al., 2009). However, other studies failed to support the decomposed model because they found no decade-unit compatibility effect, both in number comparison (Ganor-Stern, Pinhas, & Tzelgov, 2009; Zhang & Wang, 2005; Zhou, Chen, Chen, & Dong, 2008) and when using semantic priming paradigms (Reynvoet & Brysbaert, 1999; Reynvoet, Brysbaert, & Fias, 2002).

Holistic and decomposed representations are not necessarily mutually exclusive. Number comparison studies suggest that the decade-unit compatibility effect is found when the numbers are presented simultaneously but not when they are presented sequentially, suggesting that subjects can adopt either a holistic or a decomposed strategy according to task demands (Ganor-Stern et al., 2009; Zhang & Wang, 2005; Zhou et al., 2008; but see Moeller, Nuerk, & Willmes, 2009 for an alternative explanation that conforms to a decomposed approach).

1.2. Compressive vs. linear quantity representation

Much evidence shows that the internal quantity representation is tightly related with space, and that quantities are represented along a mental number line: in left-to-right readers at least, the magnitude of numbers influences manual responses made in the right or left side of space (Dehaene, Bossini, & Giraux, 1993; Shaki, Fischer, & Petrusic, 2009), eye gaze direction (Loetscher, Bockisch, Nicholls, & Brugger, 2010; Ruiz Fernández, Rahona, Hervás, Vázquez, & Ulrich, 2011), and the direction to which spatial attention is shifted (Fischer, Castel, Dodd, & Pratt, 2003). Furthermore, magnitude was shown to be encoded not only categorically as "small" or "large", but in a continuous manner (Ishihara et al., 2006).

A common paradigm to explore the quantity representation consists in analyzing how individuals map numbers to positions on a number line. How subjects map numbers to space is assumed to reflect, at least in part, the structure of the mental number line, and hence of the quantity representation (Barth & Paladino, 2011; Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Booth & Siegler, 2006; Cappelletti, Kopelman, Morton, & Butterworth, 2005; Siegler & Booth, 2004; Siegler & Opfer, 2003; von Aster, 2000; but see Núñez, Cooperrider, & Wassmann, 2012). Number-to-position studies showed that young children initially map quantities using a compressive scale that resembles a log function, but this changes into a linear encoding during the first years of school (Berteletti et al., 2010; Booth & Siegler, 2006; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003). The log-to-linear shift was hypothesized to result from education, and indeed compressive encoding was found in uneducated non-western adults but linear encoding was found in American adults (Dehaene, Izard, Spelke, & Pica, 2008). Interestingly, a compressive quantity scale can still be found in educated adults in other tasks that tap an implicit level of representation: inattentive mapping of non-symbolic quantities to position along a line (Anobile, Cicchini, & Burr, 2012), quantity estimation with non-spatial responses (Núñez, Doan, & Nikoulina, 2011), price estimation (Dehaene & Marques, 2002), number bisection (Lourenco & Longo, 2009), and randomness judgment for sequences of numbers (Banks & Coleman, 1981; Viarouge, Hubbard, Dehaene, & Sackur, 2010).