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The limitations of model fit in comparing the bi-factor versus higher-order models of human cognitive ability structure

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ABSTRACT

We addressed the question of whether the bi-factor or higher-order model is the more appropriate model of human cognitive ability structure. In previously published nested confirmatory factor analyses, the bi-factor model tended to be better fitting than the higher-order model; however, these studies did not consider a possible inherent statistical bias favouring the fit of the bi-factor model. In our own analyses and consistent with previous empirical results, the bi-factor model was also better fitting than the higher-order model. However, simulation results suggested that the comparison of bi-factor and higher-order models is substantially biased in favour of the bi-factor model when, as is commonly the case in CFA analyses, there is unmodelled complexity. These results suggest that decisions as to which model to adopt either as a substantive description of human cognitive ability structure or as a measurement model in empirical analyses should not rely on which is better fitting.

1. Introduction

Historically, a key interest in cognitive ability research has been the determination of the structure of human cognitive ability. Early debates were concerned with whether ability should be described in terms of a single general factor (e.g. Spearman, 1927) or multiple specific ability factors (e.g. Guilford, 1967; Thurstone, 1938) but these opposing theoretical perspectives eventually found conciliation in the adoption of models with multiple strata of ability factors ranging in breadth from specific to general (Gustafsson, 2001; Mackintosh, 2011). Such multi-strata models of cognitive ability structure are now well established, are reflected in contemporary theoretical models of ability structure, and have received extensive empirical support from exploratory and confirmatory factor analyses (e.g. Carroll, 1993; Johnson & Bouchard, 2005; McGrew, 2009; Vernon, 1964).

Although, implicitly, it is often assumed in using these multi-strata models that *g* is super-ordinate to more specific abilities, this is only one hypothesis about how multiple ability factors are related to cognitive performance and other models may explain the data equally well, or better. It is useful to discuss these hypotheses in terms of psychometric models of ability structure because these models facilitate the operationalisation and empirical testing of such hypotheses in a mathematically precise and falsifiable framework (Johnson & Bouchard, 2005; Vrieze, 2012).

The most commonly used psychometric model of human cognitive ability is the higher-order model, an example of which is shown in the top panel of Fig. 1. Implicit in the model are several assumptions about human cognitive ability structure, beyond the base assumption that both a *g* factor and specific ability factors play roles in cognitive performance. The model also represents the assumption that the effects of *g* on observed subtests are completely mediated by lower-order, more specific abilities. This means that *g* is assumed not to be directly involved in cognitive performance, and its effects on cognitive performance are realised only through its influences

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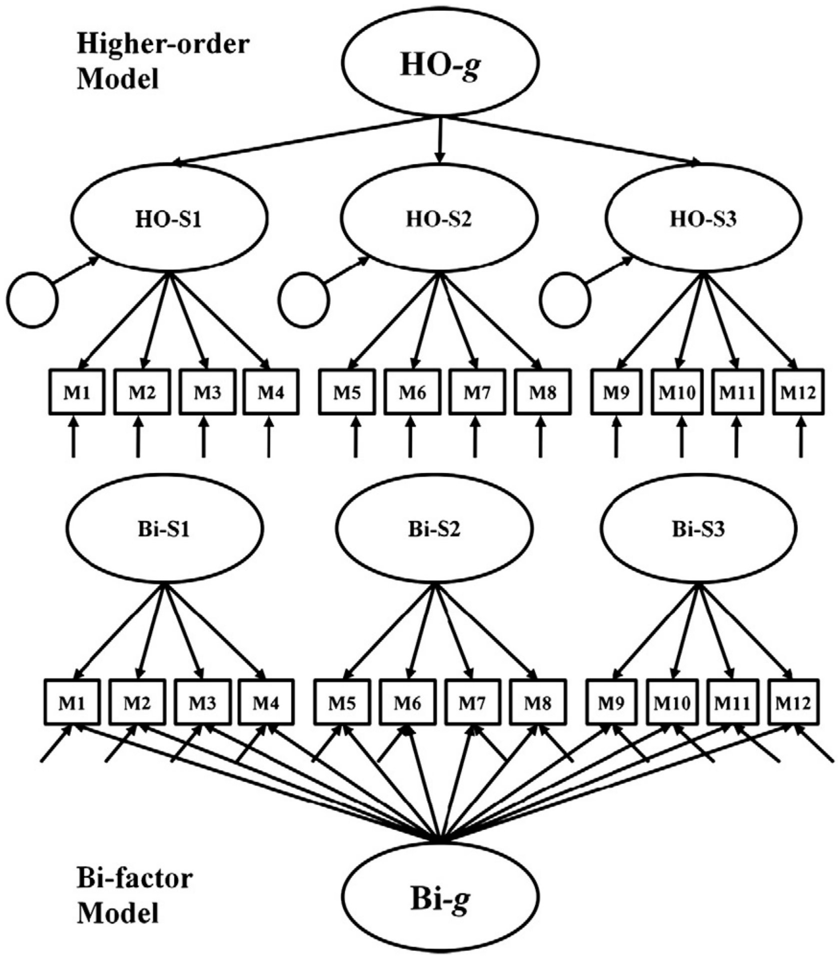

Fig. 1. Example of higher-order and bi-factor models. The basic structures of the higher-order model (top panel) and the bi-factor model (bottom panel). In the higher-order model, the associations with *g* on observed subtests (M1–M12) are mediated by the specific ability factors (HO-S1, HO-S2 and HO-S3). In the bi-factor model the associations with *g* on observed subtests are direct and independent of the effects of the specific ability factors (Bi-S1, Bi-S2 and Bi-S3) on observed subtests.

on the more specific abilities which are directly involved in cognitive performance.

Another model which can equally represent the existence of both a general factor and group factors, and thus, a plausible alternative to the higher-order model, is the bi-factor model. The higher-order model is mathematically more constrained than, and nested within the bi-factor model. An example of the bi-factor model is shown in the bottom panel of Fig. 1 (Yung, Thissen, & McLeod, 1999). The bi-factor model represents some differing assumptions about the structure of human cognitive ability. In contrast to the higher-order model, the bi-factor model reflects the assumption that the associations of *g* with observed cognitive performance are direct and independent of the associations of specific abilities with cognitive performance. These specific abilities are assumed to reflect narrower abilities such as 'Verbal' or 'Spatial' ability (e.g. Brunner, Nagy, & Wilhelm, 2012) that are independent of *g*.

From a substantive perspective, an empirical comparison of the higher-order and bi-factor model may reveal which model best approximates the 'true' structure of human cognitive ability structure, similar to historical studies that have established that a hierarchical model with both *g* and specific abilities is a more appropriate description of human cognitive ability structure than a one factor *g* model (e.g. Gustafsson, 2001). On a more pragmatic level, the measurement of *g* and specific abilities in empirical studies should utilise the best available statistical operationalisations of the constructs and the bi-factor and higher-order model are competing statistical operationalisations. Model selection issues such as those above typically rely on comparing the fits of competing models with the better fitting model being accepted as the more appropriate substantive description and/or practical operationalisation for a construct or constructs (e.g. Vrieze, 2012).

The bi-factor and higher-order models have previously been compared in this way, in a number of previous empirical studies, which aimed to determine which of the two models can best account for observed correlation amongst subtests. Such studies have used nested models confirmatory factor analysis (CFA) to compare the global fit of the two models. This is made possible by the fact that the class of higher-order CFA