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Three-dimensional arm movements at constant equi-affine speed

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ABSTRACT

It has long been acknowledged that planar hand drawing movements conform to a relationship between movement speed and shape, such that movement speed is inversely proportional to the curvature to the power of one-third. Previous literature has detailed potential explanations for the power law's existence as well as systematic deviations from it. However, the case of speed–shape relations for three-dimensional (3D) drawing movements has remained largely unstudied. In this paper we first derive a generalization of the planar power law to 3D movements, which is based on the principle that this power law implies motion at constant equi-affine speed. This generalization results in a 3D power law where speed is inversely related to the one-third power of the curvature multiplied by the one-sixth power of the torsion. Next, we present data from human 3D scribbling movements, and compare the obtained speed–shape relation to that predicted by the 3D power law. Our results indicate that the introduction of the torsion term into the 3D power law accounts for significantly more of the variance in speed–shape relations of the movement data and that the obtained exponents are very close to the predicted values.

1. Introduction

The characterization of spatial and temporal properties of human movement has long been one avenue of investigation into human movement production. This approach is important because characterization of movements not only allows one to form qualitative models of the mechanisms behind movement production, but also permits close examination of the predictions of quantitative models of motion production. In this paper we investigate a characteristic of

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human movement – the speed of drawing movement is related to its geometry. This empirical relationship has been described in general terms as an inverse relationship between movement speed and curvature (Binet and Courtier, 1893; Jack, 1895; Abend et al., 1982). More specifically, for planar drawing movements, it was termed the *two-thirds power law* (denoted 2/3-PL below), relating speed to curvature (Lacquaniti et al., 1983; Viviani and Cenzato, 1985), or more formally:

$$v = \alpha\kappa^{-1/3}. \tag{1}$$

Here v is the movement speed, κ is the curvature of the path, and α is some constant, termed the *velocity gain factor* (or *speed gain factor*). The law acquired its name from its expression in terms of angular speed, A, i.e., $A = \alpha\kappa^{2/3}$.

As detailed below, various types of drawing movements have been demonstrated to obey the power law (e.g., Viviani and McCollum, 1983; Viviani and Cenzato, 1985). This power law has also been suggested to have a role in locomotion (Vieilledent et al., 2001; Hicheur et al., 2005) and motion perception (Viviani and Stucchi, 1989, 1992; Levit-Binnun et al., 2006). Moreover, possibly more importantly, it has been related to neural coding of motor commands during various types of drawing movements according to the population vector model (Georgopoulos et al., 1982, 1986, 1988; Schwartz, 1992, 1993, 1994; Moran and Schwartz, 1999; Schwartz and Moran, 1999, 2000) as well as to proprioceptive feedback (Viviani et al., 1997; Albert et al., 2005).

One incomplete aspect of these relations between kinematics and geometry is the question of whether speed is related to geometrical features of three-dimensional (3D) movements such as curvature and torsion. Previous research has not revealed an obvious relationship between speed and torsion (Morasso, 1983). Moreover, there is evidence that the 2/3-PL describing planar movements does not satisfactorily describe 3D movements (Pollick and Ishimura, 1996; Schaal and Sternad, 2001). In the following, we propose a mathematically derived novel relationship between speed and 3D geometry for drawing movements, evaluate it empirically on spatial scribbling hand movements, and compare it to the 2/3-PL. By scribbling movements we mean spontaneous unconstrained movements of the hand, where the subject is guided by no template or instructions but those of her or his own wish (see Fig. 2 for an example of the types of scribbles produced in this experiment).

The starting point for this research is a recent theoretical description of the 2/3-PL, which showed that the covariation of speed and curvature that this power law describes in planar drawing motion is consistent with the mathematical interpretation of motion at constant equi-affine speed[2] (Flash and Handzel, 1996, 2007; Pollick and Sapiro, 1997; Handzel and Flash, 1999).

This interpretation has implications for the representation of figural forms (see next section). But, more importantly for us here, the conditions for motion at constant equi-affine speed in two dimensions can be generalized to 3D movements, where they entail a new relationship between speed and geometry (Pollick et al., 1997). The details of this derivation are presented below (in the Appendix) and provide a predicted relationship: that the speed (v) of a 3D movement is proportional to the inverse of the product of the curvature (κ) raised to the 1/3 power and the absolute value of the torsion (τ) raised to the 1/6 power.[3] More formally:

$$v = \alpha\kappa^{-1/3}|\tau|^{-1/6} = \alpha\left(\kappa^2|\tau|\right)^{-1/6}, \tag{2}$$

where α is once again a constant, termed the velocity gain factor (or speed gain factor). We name this relationship between the speed, curvature and torsion the *one-sixth power law* (usually designated 1/6-PL below).

The empirical thrust of this paper is to examine unconstrained self-paced spatial scribbling movements, to see whether they conform to this new 3D power law. We will also compare between the adherence of our data to both of the above power laws.

1.1. *Planar movement*

It has been found that for production of continuous planar hand movements such as drawing movements, the relationship between speed and curvature is rather well described by a power law where speed is proportional to the curvature to the power of –1/3, or equivalently to the 1/3 power of the radius of curvature (Lacquaniti et al., 1983). This power law has been found in a variety of drawing tasks (Viviani and McCollum, 1983; Viviani and Cenzato, 1985; Wann et al., 1988; Massey et al., 1992; Viviani and Flash, 1995), and has been shown to evolve with the development of drawing skills (Viviani and Schneider, 1991). In addition to describing the production of planar drawing movements, the visual perception of planar form (Viviani and Stucchi, 1989) and movement uniformity (Viviani and Stucchi, 1992; Levit-Binnun et al., 2006) were both found to be influenced by deviations from the 2/3-PL. Why one would expect the 2/3-PL relation for perception, let alone production of human movement, is an open question that has received considerable attention, and is discussed directly below.

1.2. *The 2/3-PL as an emergent property*

The tendency of human drawing movements to obey the power law may reflect the internal neural representation of movement by the central nervous system (CNS) (Schwartz,

[2] ''Equi-affine transformations'' are a special case of the more general ''affine transformations'', in which the determinant of the affine transformation matrix is strictly 1. See the Appendix for more information.

[3] By definition, the curvature is the normal of the unit tangent vector to the curve (Oprea, 1997) and is hence non-negative. Intuitively it is the amount of local deviation from straightness of the curve. Torsion, on the other hand, is the negative of the dot-product of the curve's principle normal vector and the derivative of the binormal vector, and can thus take positive or negative values (Oprea, 1997). Intuitively it is the amount of local deviation from planarity. In more technical terms, it is the signed rate of change of the osculating plane, where the sign is determined by the direction of movement. In the setting of our power law we are interested in the rate of change of the osculating plane, rather than in its direction. We therefore use the absolute value, or magnitude, of torsion.