Neuropsychologia 49 (2011) 3629–3635

Contents lists available at SciVerse ScienceDirect

Neuropsychologia

journal homepage: www.elsevier.com/locate/neuropsychologia

On the dependency of division on multiplication: Selective loss for conceptual knowledge of multiplication

Annalena Venneri[a,b,*], Carlo Semenza[c,b]

[a] Department of Neuroscience, University of Sheffield, and Sheffield Teaching Hospital, Sheffield, UK
[b] San Camillo Hospital (IRCCS), Venice, Italy
[c] Department of Neuroscience, University of Padua, Italy

ARTICLE INFO

Article history:
Received 5 January 2011
Received in revised form 16 August 2011
Accepted 6 September 2011
Available online 17 September 2011

Keywords:
Multiplication
Division
Dementia
Dyscalculia

ABSTRACT

This study reports the case of a patient (AG) whose main calculation problem was with multiplication that was severely affected at all levels including that of conceptual knowledge. In contrast AG's problems with addition, subtraction and division were much less conspicuous and just involved the use of procedures. For all these latter operations, and for division in particular, conceptual knowledge was spared. Despite his procedural problems, AG in fact was fully aware that division consists of finding out how many times a given quantity is contained in another quantity. Thus while he was able to reach the correct results for division by subtracting the divisor from the dividend, he never used the reverse strategy to complete multiplication operations, i.e. add the number for as many times as the multiplier to complete the operation, and showed no awareness that this was what multiplying means, even when explicit suggestions were made by the examiner.

The existence of AG's case could not be expected on the basis of theories holding that division depends on multiplication and that is not separately represented in semantic memory. It follows, therefore, that the extent to which division depends on multiplication needs to be reconsidered.

1. Introduction

This neuropsychological study represents a contribution to the understanding of the relationship between multiplication and division. Does the latter depend on the former? The extent to which division and multiplication are interdependent has been first specifically and extensively investigated in non brain-damaged populations within experimental as well as developmental psychology. One possibility is that corresponding problems in the two operations are represented and retrieved as independent facts in memory. Supporting this hypothesis, earlier studies by Rickard and co-workers found little transfer of practice between multiplication and division (Rickard & Bourne, 1996; Rickard, Healy, & Bourne, 1994). If transfer of practice measures strengthening of retrieval processes, this negative result implies that performance of corresponding division and multiplication does not depend on common retrieval processes. On the basis of these findings, Rickard and co-workers proposed a model (extended in Rickard, 2005) that mainly suggests that problems that have the same operands are represented by a common node. For each triplet of numbers that are related by complementary multiplication and division problems (e.g. 4, 7, 28), there are three independent fact representations in memory: (4, 7, X) $\rightarrow$ 28; (28/7) $\rightarrow$ 4; and (28/4) $\rightarrow$ 7. Large division problems may not be directly retrieved from memory representations, however, and may be reframed as the corresponding multiplication to obtain the answer (Rickard, 2005). Similar conclusions were reached by other studies. LeFevre and Morris (1999) found that when people solved simple multiplications (e.g. 8×7) and the corresponding division problems (e.g. 56/8), latency patterns and error rates were closely related across operations. The solution of division problems, however, facilitated the solution of multiplication problems more than the reverse. Moreover, large division problems (e.g. 56/8) were "re-cast" as multiplication problems, i.e. in the form of $8 \times ? = 56$. While these results support the hypothesis that at least simple multiplication and division are stored in separate mental representations, the solution of difficult division problems would sometimes involve access to multiplication (on this point see also Mauro, LeFevre, & Morris, 2003). Campbell (1997, 1999) also found evidence that performance of simple division involved multiplication. In particular he found that reaction times were highly correlated for corresponding division and multiplication problems (Campbell, 1997); furthermore, error characteristics indicated parallel retrieval structures. However, since he found division to multiplication priming but not multiplication to division priming, Campbell (1997) suggested that, while the performance of corresponding division and multiplication problems involves separate memory representations,

* Corresponding author at: Department of Neuroscience, University of Sheffield, Medical School, Beech Hill Road, Sheffield S10 2RX, UK.
E-mail address: a.venneri@sheffield.ac.uk (A. Venneri).

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doi:10.1016/j.neuropsychologia.2011.09.017

participants in his experiment frequently used multiplication as a check after direct retrieval of the quotient. Thus, Campbell (1997) outlined two possible alternative hypotheses for the representation of multiplication and division knowledge. In the first "*common representation*" hypothesis, he suggested that a single network exists that is accessed differentially according to the specified operation. In the second "*independent representation*" hypothesis, he suggested that two separate mental networks exist, one for each operation. It must be noticed that children apparently rely initially on their knowledge of addition and multiplication to solve division (Geary, 1994; Siegler, 1988).

In summary, the above reported studies do not provide univocal indications about whether division depends on multiplication. Moreover, most of these studies concerned arithmetical facts and, in particular, do not tell anything about the representation of division and multiplication in conceptual knowledge. Evidence from neuropsychology and cognitive neuroscience may help addressing and clarifying these issues.

After brain damage, mathematical operations may be selectively impaired or spared for a number of reasons and impairment or sparing of a single type of operation may, therefore, result from deficits affecting different aspects of processing (see for reviews Butterworth, 1999; Cipolotti & van Harskamp, 2001; Noël, 2001; Semenza, Granà, & Girelli, 2006; Semenza, 2008; Ward, 2010). Thus, for example, multiplication may be affected by loss of tables, loss of specific procedures, etc. Additionally, loss of multiplication procedures, i.e. the sequences of steps necessary to perform multi-digit operations, may result either from faulty knowledge of some specific aspect of the operation (i.e. carrying, or the spatial lay-out required for a complex written operation) or from the inability to monitor intermediate steps to the end (Granà, Hofer, & Semenza, 2006; Semenza, Miceli, & Girelli, 1997).

Within this picture, neuropsychological reports on division are relatively scarce. Clinical investigations of mathematical skills often ignore the assessment of division altogether; likewise, single case reports of dissociation among operations most often do not even consider division (see, for this specific issue, a review by Cipolotti & van Harskamp, 2001). One (intuitive) reason is that division is generally regarded as the hardest of the four basic operations, the one that many adults have forgotten how to perform. It should not come as a surprise, therefore, if, following brain damage, division appears impaired, for whatever reason, including the fact that brain injured people may simply lack adequate resources to carry out such a complex task. Neuropsychological studies on division, however, may be more revealing than generally thought. In particular they may highlight the relation between division and multiplication.

Some data favour the idea of independent stores and processing. Thus Cipolotti and de Lacy Costello (1995) reported the case of a patient (CB) with a selective deficit in performing division. They argued (see also Cipolotti & van Harskamp, 2001) that this finding is more consistent with models whereby each operation has its distinct routine or store (e.g. Dagenbach & McCloskey, 1992), than with a model like that proposed by Dehaene and Cohen (1995), that, although not overtly claiming complete interdependency, makes no explicit assumptions about completely independent memory stores for different arithmetic operations. However, selective impairments of division can easily be explained with the assumption that division is just harder than other operations. In fact, in schools divisions are taught last and less systematically than other operations (Lucangeli, Tressoldi, Bendotti, Bonanomi, & Siegel, 2003). While, for instance, multiplication tables are taught for years, a comparable task is later never given to train children to acquire skills in division.

The idea that divisions are separately stored in long-term memory and independently processed finds little support in other neuropsychological findings that have clearly shown how divisions are answered by reference to related multiplication problems (e.g. Delazer et al., 2004; Girelli, Delazer, Semenza, & Denes, 1996; Hittmair-Delazer, Semenza, & Denes, 1994). Thus, for example, patient BE (Hittmair-Delazer et al., 1994) presented parallel loss and preservation for corresponding multiplication and division problems. When rehabilitated for multiplication, the benefit transferred to division (see Girelli et al., 1996, for similar results in other patients). These data led the authors to conclude that BE's division performance depended on the availability of the corresponding multiplication fact (note that BE's conceptual knowledge was intact, as shown by Hittmair-Delazer et al., 1994).

Congruently with these latter findings, dissociating impairments between the operations of division and multiplication, with sparing of the former, are deemed impossible as division is said to be dependent on intact multiplication abilities. Indeed, thus far, this type of dissociation has never been described, and the general intuition is that, for most people, carrying out division does depend on knowing multiplication (Butterworth, 1999). Recently a single case has been reported who showed a combination of calculation procedure impairments that left the procedures involved in division intact but affected all other operations (Chiarelli, Menichelli, Zadini, & Semenza, 2011). Conceptual knowledge was spared for both multiplication and division, however. The nature of this deficit, therefore, does not provide evidence in support of the conceptual independence of division from multiplication. Thus, to date no evidence has been provided that might support the possibility of a selective loss of conceptual knowledge for multiplication in the presence of preserved conceptual knowledge for division, in a context of otherwise intact semantic and calculation abilities.

Neuroimaging studies do not help clarifying the issue. An fMRI study by Ischebeck, Zamarian, Schocke, & Delazer (2009) compared division problems related to multiplication (transfer condition) to unrelated division problems (no-transfer condition). Overall, no significant differences were observed. A significant transfer effect was found within the left angular gyrus, a brain area involved in the retrieval of arithmetical facts. According to the authors this pattern of findings seemed to suggest that newly acquired multiplication fact knowledge may be recruited for the solution of unknown division problems. This fact knowledge might be either used directly in finding the solution, or used to check the correctness of the division result in a subsequent step: a decision between these two possibilities cannot be easily made. Ischebeck et al. recognized that their findings are compatible with models that posit independent fact representations for corresponding multiplication and division. Importantly, distinct inter-individual differences with regard to transfer were detected.

In summary, both investigations in normal participants and neuropsychological findings show how the relation between mental processes involved in, respectively, multiplication and division is a very complex one, and depends on the processing level required by each given task. The present study contributes to the understanding of such relation by reporting the case of a patient, AG, affected by a progressive brain disease, who in the early stage of cognitive decline showed relative sparing of conceptual knowledge for division contrasting with an extremely severe, across the board, impairment of multiplication.

2. Case report

AG[1], a highly educated man with a Bachelor degree in commerce and a Master degree in accountancy, aged 53 at time of this study, was referred aged 51, in 1996 because of difficulties with

[1] The patient's initials have been changed to protect his anonymity.