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# Multifractal analysis of wind velocity data

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ABSTRACT

We analyze the wind velocity, $v(t)$, at a location in the Patagonia, Argentina, as a function of time, measured every 10 min, uninterruptedly, over a one-year period, by taking the graph $(t, v(t))$ as a signal, and applying the tools of multifractal analysis. The multifractal spectrum $(\alpha, f(\alpha))$ of this signal, obtained by the definition of $\alpha$ and $f(\alpha)$, is strangely smooth, for it fulfills all properties of the twice-differentiable thermodynamical formalism, the theoretical algorithm devised to mimic the spectrum-by-definition (with which it may not coincide, since the latter may not even be continuous). We give arguments to conclude that the wind-spectrum is of Weierstrass type. We take a finite sample of Weierstrass functions, $W_{\lambda,s}(t)$, at regular intervals $\Delta t$, the size of each sample comparable to that of the wind signal. We take these samples as signals, and construct their multifractal spectra by definition, for different parametric values of $\lambda$ and $s$. We compare the different Weierstrass spectra with the wind spectrum, and obtain information on the Weierstrass parametric values that agree with the wind spectrum. We interpret the key points of the wind spectrum in meteorological terms. Any change, from one year to the next, in the position of such key points, or in the shape of the spectral curve—once interpreted in the corresponding meteorological terms—could be some local indicator of climate change.


## Introduction

The wind speed and direction change continuously. However, in processing wind data for wind power applications, the data are reduced to statistics, e.g. average speeds and wind roses. The distribution of wind speed over time is characterized by the Weibull function.

Work has been done on fractal analysis of wind data (Chang et al., 2012; Tijera et al., 2008; Tarquis et al., 2005, among others). In this paper we related the wind data to a theoretical model involving multifractal analysis.

## Basic notions

A planar fractal geometrical configuration is a set $F$ to which we can associate a dimension not necessarily an integer. A well known example is the Koch curve, obtained by replacing three segments by four of the same length, and iterating the process. See Fig. 1.

Such an object $F$ is said to be *self-similar*, in the sense that we iterate the same three-segments-by-four replacement process over and over, ad infinitum.

For such self-similar objects it is very easy to calculate the fractal dimension, log 4 / log 3 in the Koch case. From this Koch case, we can infer that though a continuous planar curve $F$ can be self-similar, the graph of a function cannot. It can achieve dimension larger than unity (between 1 and 2) only by two different processes of replacement, one horizontal and the other vertical, and the latter is larger than the former.

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A well known example is the graph of the *Weierstrass function*, $W_{\lambda,s}(t) = \sum_{k=0}^{\infty} \frac{\sin\lambda^k t}{\lambda^{(2-s)k}}$, $1 < s < 2$, $\lambda > 1$. Notice that, in the horizontal $t$-axis, we deal with $\frac{1}{\lambda}$ amplitudes, in the sense that $\sin\lambda^{k+1}t$, with $0 \le t \le 2\pi$, is a $\frac{1}{\lambda}$ contraction of $\sin\lambda^k t$, $0 \le t \le 2\pi$. But the vertical $W(t)$ change is given by $\frac{1}{\lambda^{2-s}}$, from $\frac{\sin\lambda^k t}{\lambda^{(2-s)k}}$ to $\frac{\sin\lambda^{k+1} t}{\lambda^{(2-s)(k+1)}}$, and $\frac{1}{\lambda^{2-s}}$ is larger than $\frac{1}{\lambda}$.

Such non-self-similar processes are called *self-affinal*.

In the general case, the fractal *box dimension* $d_B$ of an object $F$ is calculated thus: we embed the set $F$ in a grid of squares of small size $\ell$. Let $N_\ell$ be the number of squares exactly covering $F$. Then $d_B = \frac{\log N_\ell}{\log\frac{1}{\ell}}$, if $\ell$ is rather small.

A set $F$ does not, necessarily, have a regular process of replacement such as the 3-by-4-segments of the Koch case. It might be that a plurality of such replacement processes is present in an irregular object. "Natural" fractals are not, in general, endowed with a pure and unique rule of replacement. That is why they are called *multifractals*.

The decomposition of such an object into more self-similar non-intersecting subfractals, each of them with a different dimension $d_B$, is called the *multifractal spectrum* of the set $F$.

There are two ways, in the literature, to obtain a multifractal spectrum, the *spectrum-by-definition*, and the so-called *thermodynamical algorithm*.

## The spectrum-by-definition $f(\alpha)$ of a planar signal

We build the spectrum in stages. We take the signal, in our case a discrete finite sample of the graph of $W_{\lambda,s}(t)$, taken at equal consecutive

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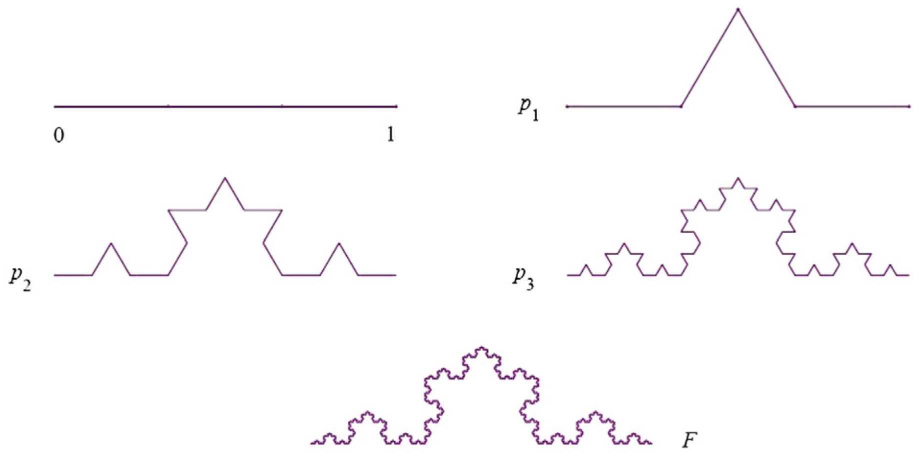


**Fig. 1.** Polygonal approximation to the von Koch curve.

intervals $\Delta t$, or the discrete graph of $v(t)$, the wind velocity as a function of time. We cover the signal with a grid of square boxes of size $\ell$; in our case, $\ell = 1/81$ throughout the paper. We enumerate the boxes $B_i$, there is a finite number of them. The probability measure, $pi_i$, of the signal in $Bi_i$ is the number of points of the discrete signal inside that box, normalized by the total number $n$ of points in the signal. In our case, $n = 128{,}000$, for all Weierstrass samples. Once we have the probabilities $\{p_i\}$, we obtain the "concentration" $\alpha_i$ of the signal in box $B_i$ as the log/log version of the "density", i.e. $\alpha_i : \frac{\log p_i}{\log \ell}$. There is an $\alpha_{min}$ and an $\alpha_{max}$. The range of $\alpha$'s, $[\alpha_{min}, \alpha_{max}]$, is now divided into disjoint and consecutive intervals $I_\alpha[\alpha, \alpha + \Delta\alpha]$ of equal length $\Delta\alpha$. $N_\alpha$ is the number of $\alpha_i$'s in $I_\alpha$, i.e. the number of those $\alpha_i \approx \alpha$. Then $f(\alpha) : \frac{\log N_\alpha}{\log \frac{1}{\ell}}$ if $\ell$ is small is the definition of the box dimension of the signal inside those boxes $B_i$ with $\alpha_i$ in $I_\alpha$. In other words: $f(\alpha)$ is the box dimension of that part of the signal with points that correspond to an $\alpha$-concentration roughly equal to $\alpha$. The total number of $\alpha_i$'s, i.e. of occupied boxes $B_i$, is distributed among those intervals of width $\Delta\alpha$.

## The thermodynamical formalism

The interval $I_\alpha$ above with a larger number $N_\alpha$ of $\alpha_i$'s is the one corresponding to the maximal value of $f(\alpha)$, i.e. *fmax*. The following algorithm $f_{th}(\alpha)$, "th" for "thermodynamical", can yield a very different situation for $f_{th}max$, as seen below.

The spectrum $f_{th}(\alpha)$ is given by the following set of equations (Falconer, 1990, Chap. 17):

$$q := f'_{th}(\alpha)$$

$$\tau(q) : \frac{\log \sum_i {p_i}^q}{\log \ell}, \ \ell \text{ has to be relatively small}$$

$$\alpha(q) = \tau'(q)$$

$$f_{th}(\alpha(q)) = q\,\alpha(q) - \tau(q)$$

$$f''_{th}(\alpha) < 0$$

where $\ell$, $\{p_i\}$, have the significance as above, and $q$ is an arbitrary parameter in $(-\infty, \infty)$, and acts as a parametric variable yielding $\alpha(q)$ and $f_{th}(\alpha(q))$.

Since $q = f'_{th}(\alpha)$, the maximum value $f_{th}max$ corresponds to $q = 0$, i.e. to $f'_{th}(\alpha) = 0$. From the set of equations above, $f_{th}(\alpha(0)) = f_{th}\max = 0\ \alpha(0) - \tau(0) = -\tau(0) = -\frac{\log \sum_i {p_i}^0}{\log \ell} = \frac{\log \sum_i 1}{\log \frac{1}{\ell}} = \frac{\log(\text{total number of boxes occupied by the signal})}{\log \frac{1}{\text{size of each box}}}$, which is the box dimension of the signal—all of it: in the numerator we have "all" $\alpha_i$'s, whereas in *fmax*, we have only the largest $N_\alpha$. The spectrum-by-definition ascribes to *fmax* only the $\alpha_i$'s in a certain (small) interval of length $\Delta\alpha$, so, sometimes, $f(\alpha)$ is smaller than its thermodynamical counterpart. This discrepancy disappears when the number of boxes

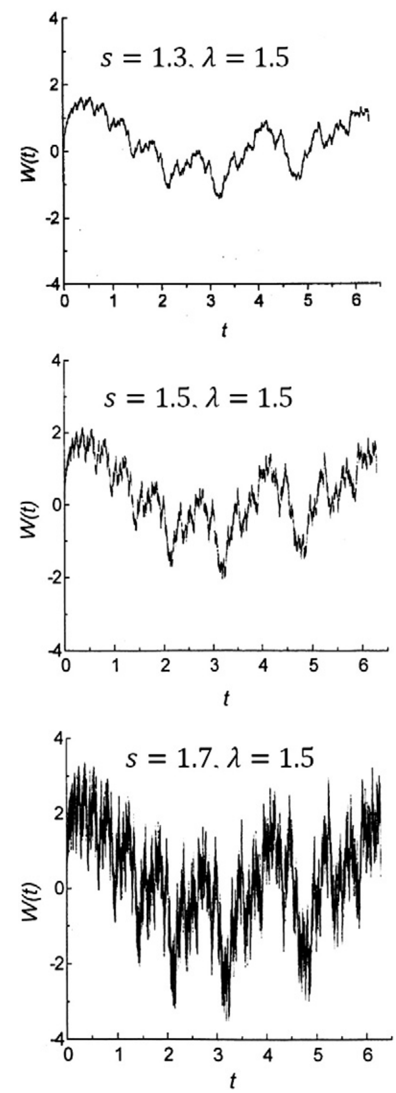


**Fig. 2.** Weierstrass function, $W_{\lambda,s}(t)$ for several values of $s$ (and $\lambda = 1.5$).