Available online at www.sciencedirect.com
SciVerse ScienceDirect
Journal of Economic Theory 148 (2013) 1841–1861
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Incentive compatibility and differentiability: New results and classic applications

George J. Mailath [a,*], Ernst-Ludwig von Thadden [b]

[a] *Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104-6297, USA*
[b] *Department of Economics, Universität Mannheim, D-68131 Mannheim, Germany*

Received 9 November 2011; final version received 26 January 2013; accepted 15 February 2013
Available online 26 July 2013

Abstract

We provide several generalizations of Mailath's (1987) [9] result that in games of asymmetric information with a continuum of types incentive compatibility plus separation implies differentiability of the informed agent's strategy. The new results extend the theory to classic models in finance such as Leland and Pyle (1977) [8], Glosten (1989) [4], and DeMarzo and Duffie (1999) [3], that were not previously covered.

JEL classification: C60; C73; D82; D83; G14

Keywords: Adverse selection; Separation; Differentiable strategies; Incentive-compatibility

1. Introduction

In many problems of asymmetric information, one agent has private information upon which she bases her actions, and uninformed agents act based on inferences from these actions. Because the informed agent can reveal information through her actions, she chooses her actions strategically. If the informed agent's action as a function of her private information is one-to-one, then her strategy is said to be *separating* and her actions completely reveal her private information.

If the agent's private information (her "type") is given by a continuously distributed real-valued random variable, incentive-compatible separating strategies in such interactions can easily

* Corresponding author. Fax: +1 215 573 2057.
E-mail addresses: gmailath@econ.upenn.edu (G.J. Mailath), vthadden@uni-mannheim.de (E.-L. von Thadden).

0022-0531/$ – see front matter
http://dx.doi.org/10.1016/j.jet.2013.07.004

be characterized by a differential equation, *if* the strategy is known to be differentiable. But exactly because the strategy is not known, differentiability cannot be taken for granted. This poses a serious problem for the determination and uniqueness of equilibrium.

In many cases, however, differentiability is an implication of incentive-compatibility. For a large class of signaling games and related settings, Mailath [9] has shown that any incentive-compatible separating strategy of the informed agent must be differentiable and hence satisfy the standard differential equation. Unfortunately, the assumptions in [9] rule out many important applications. In particular, as we describe below, they do not cover the models of Leland and Pyle [8], Glosten [4], and DeMarzo and Duffie [3] that are at the core of modern theories of corporate finance and market microstructure.

In this paper, we provide appropriate generalizations of Mailath [9] to cover these models. The new results can be grouped into two categories. First, we provide new sufficient conditions on payoff functions for differentiability to be implied by incentive-compatibility. For example, we show that differentiability can obtain even in linear models, which are not covered by Mailath [9]. This extends the analysis to those models in corporate finance or Industrial Organization that use risk-neutrality, where the classic first-order conditions of expected utility theory do not apply.

The second category of results refers to the underlying type and action sets. We show that the original and our new sufficiency conditions extend to non-compact (in particular, unbounded) type sets, and to bounded action sets (in [9] the action set is $\mathbb{R}$). This is important for two reasons. First, many applications, in particular in finance, naturally involve unbounded type sets, for example, when using normally distributed returns, or bounded action sets, for example when short-selling is not allowed. Second, working with arbitrary type sets makes it possible to apply the sufficient conditions locally instead of globally. For this, one simply considers subintervals of the original type set, when the sufficient conditions do not hold globally (because, for example, a derivative vanishes somewhere), but do hold locally (because the derivative cannot vanish everywhere). We provide an example in which global differentiability can be shown by "patching together" local arguments.

Our interest in the differentiability of separating strategies leads us to study environments with differentiable payoffs. The standard theory of ordinary differential equations, together with the usual boundary conditions then implies the uniqueness of separating equilibrium strategies. This uniqueness also holds more generally in environments with continuous payoffs. See [13] for global conditions that guarantee uniqueness of the separating equilibrium when there is a continuum of types.

2. The model

An informed agent knows the state of nature $\omega \in \Omega \subset \mathbb{R}$ and one or more uninformed agents react to the informed agent's action $x \in \mathcal{X} \subset \mathbb{R}$ on the basis of inferences drawn from x about ω. The sets Ω and $\mathcal{X}$ are intervals; they may be bounded or unbounded, and we do not require the intervals to be open or closed. For our purposes, this interaction can be summarized by the $\mathcal{C}^2$ function

$$V : \Omega^2 \times \mathcal{X} \longrightarrow \mathbb{R},$$
$$(\omega, \hat{\omega}, x) \longmapsto V(\omega, \hat{\omega}, x), \tag{1}$$