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Coordination failure in repeated games with private monitoring

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Abstract

Players coordinate continuation play in repeated games with public monitoring. We investigate the robustness of such equilibrium behavior with respect to ex-ante small private-monitoring perturbations. We show that with full support of public signals, no perfect public equilibrium is robust if it induces a "regular" 2×2 coordination game in the continuation play. This regularity condition is violated in all belief-free equilibria. Indeed, with an individual full rank condition, every interior belief-free equilibrium is robust. We also analyze block belief-free equilibria and point out that the notion of robustness is sensitive to whether we allow for uninterpretable signals.

JEL classification: C72; C73; D82; D83

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1. Introduction

In repeated games with public monitoring, the players have common knowledge about past signals and coordinate continuation strategies contingent on the public histories of past signals. In many cases, however, players observe noisy signals that are slightly different for different players. The purpose of this paper is to investigate whether and how equilibrium construction

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in public-monitoring repeated games depends on the common knowledge assumption about past signals. More specifically, we investigate if players can coordinate approximately in the same way if the monitoring structure is perturbed so that there is no longer common knowledge about past signals.

We formalize the question about coordination as follows. Fix a repeated game with public monitoring. Perturb the game so that players observe noisy private signals. Each player has an "interpretation" of signals, which is a function that maps her private signals to public signals. A private-monitoring structure is close to the public-monitoring structure under a profile of interpretations if, conditional on each action profile, the probability that all players observe private signals interpreted as a common public signal y under private monitoring is close to the probability that they observe y under public monitoring. Note that we measure proximity between the two monitoring structures only in the ex-ante sense, and we do not impose any restriction on a player's interim beliefs about the opponents' signals conditional on her own signals. For example, Hörner and Olszewski [16] analyze repeated games with almost-perfect monitoring, which is close to perfect monitoring in the ex-ante sense. For another example, consider the Cournot oligopoly game where firms produce almost homogeneous outputs, and "market price" θ is drawn from a continuous distribution. Each firm i does not observe θ directly, but observes a firm-specific price:

$$\omega_i = \theta + \varepsilon_i.$$

This monitoring structure is close to the public monitoring where firms observe binary public signals about high or low prices, $\{\theta \geqslant \bar{\theta}\}$ or $\{\theta < \bar{\theta}\}$, as long as noise terms ε_i's concentrate around 0.[1]

Given the above class of perturbations, we say that an equilibrium of a public-monitoring repeated game is robust to private-monitoring perturbations if, for every private-monitoring perturbation close to the original public-monitoring structure under a profile of interpretations, there exists an equilibrium of the perturbed game close to the translation of the original equilibrium into private strategies via the same or similar profile of interpretations.

In two-player games with full support of public signals, we show that no perfect public equilibrium (henceforth PPE) is robust to private-monitoring perturbations under a certain regularity condition that excludes belief-free equilibria and in particular, repetitions of static Nash equilibria (Theorem 1). The analysis of PPE in games with public monitoring has flourished since Abreu, Pearce, and Stacchetti [1] and Fudenberg, Levine, and Maskin [13]. Theorem 1 reveals that such analysis depends critically on the common knowledge assumption regarding past histories. Except for PPEs that violate the regularity condition, no PPE can be translated into equilibria in all perturbed games; either (i) a perturbed game has no equilibrium that is close to PPEs or (ii) the way to translate a PPE depends on fine details of perturbations.

We prove Theorem 1 as follows. Suppose, for simplicity, that there are two public signals $\bar{y}$ and $\underline{y}$. Take any PPE, and imagine we are at the beginning of period 2. By the definition of PPE, it is optimal for each player to follow the continuation strategy after observing $\bar{y}$ (resp. $\underline{y}$) if the opponent also takes the continuation strategy after observing $\bar{y}$ (resp. $\underline{y}$). Now perturb the monitoring structure. At the beginning of period 2, each player chooses an interpretation of

[1] Mailath and Morris [20,21] analyze a closely related notion of almost-public monitoring. They, however, exclude almost-perfect monitoring by assuming full support of the original monitoring structure. They also exclude the Cournot example because their notion of closeness measures distances between two monitoring structures from the interim perspective. See Section 1.1.