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Bayesian serial cost sharing

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Abstract

Shared productive facilities often entail incomplete information regarding attributes such as production costs, production quality, user lists, user preferences, etc. We formulate the notion of Bayesian cost share games to model such environments and find that many of the desirable features the serial cost-sharing rule enjoys in nonstochastic settings also extend to Bayesian frameworks. We also introduce the concept of normal goods to cost-sharing games and demonstrate that normality further strengthens the serial rule's appeal.

Keywords: Bayesian games; Cost allocation; Incomplete information

JEL classification: C71; D61; D82

1. Introduction

The shared use of productive facilities is an integral feature of many economic environments, examples of which may range from high-tech telecommunications satellites to low-tech irrigation ditches. A critical issue that naturally arises in such settings is how best to allocate the production costs induced by this shared use. While this question has received considerable attention (see Moulin, 2002; Thomson, in press for extensive literature reviews), previous studies have not addressed the pervasive uncertainty that

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cooperative ventures often entail. For instance, production costs and output quality may vary unpredictably as a consequence of untested technology, idiosyncratic mechanical failures, or worker error. The fact that people and businesses frequently move, expire, or adopt alternative sources for the goods/services they desire may lead to the emergence of uncertainty in regards to the numbers, identities, and preferences of users. Further complicating matters, players may also be asymmetrically endowed with incomplete information regarding the nature of uncertainty. We introduce the concept of a Bayesian cost-sharing game to model these elements of realism and examine their impact on serial cost sharing.

The serial cost-sharing rule, first introduced by Shenker (1990), can be thought of as the cost covering price mechanism in which the price assigned to each demand is independent of all larger demands. Moulin and Shenker (1992) demonstrated that this rule is endowed with outstanding incentive properties, an assessment that has been reaffirmed by subsequent theoretical and applied studies. Our results suggest that the serial rule's allure continues to be strong, albeit slightly dampened by the presence of uncertainty. In particular, we find that whenever utilities are concave and costs are strictly convex, the implementation of serial cost sharing within uncertain environments will induce agent dominance solvable structures, i.e., the recursive elimination of strictly dominated state contingent demands converges to the unique equilibrium. As is the case under certainty, serial cost sharing is shown to be the only sharing mechanism to enjoy this property. We also find that equilibria of Bayesian serial cost sharing are resistant to defections of agent coalitions, that is, coalitions of player/information types. This too is shown to be a defining characteristic of the serial mechanism. Bayesian serial cost-sharing equilibria need not, however, be coalitionally stable in the ex ante player sense (prior to the endowment of private information), as opportunities may exist for player coalitions to increase each member's expected payoff through compromises forged across states of nature.

As noted above, one appealing feature of the serial cost-sharing rule is that its implementation induces an agent dominance solvable noncooperative game. The process of dominance solvability may, however, require infinitely many iterations of dominated strategy elimination before the equilibrium is converged upon. This implicitly suggests a degree of "hyper-rationality" on the part of agents/players that may undermine claims that players necessarily gravitate to equilibrium when left to their own devices. Confidence in the equilibrium's value as a predictive outcome may be bolstered if it can be reached through a finite elimination process. We find that this is indeed the case for an important class of cost share games that we introduce to the literature—those for which increases in "disposable income" have a nondecreasing effect on the demands agents place on the shared facility; that is, facility output is a normal good. This result applies irrespective of whether stochastic elements are involved.

To the best of our knowledge, our paper is the first to formally examine cost-sharing games in a Bayesian framework. Our paper is *not*, however, the first to examine the serial mechanism as a means for sharing costs and/or resources. The serial rule and its variants have been examined in a variety of contexts including: public good applications (Moulin, 1994), personalized demands and independence of ordinal transformations (Sprumont, 1998), multiproduct extensions (Koster et al., 1998), increasing returns to scale (de Frutos,