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A multi-agent dynamic model based on different kinds of bequests

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ABSTRACT

We investigate how wealth transfer that happens at the end of an agent's life affects its final distribution based on a multi-agent dynamic model. We discuss two kinds of wealth transfers: to a single agent and to charities. The first kind of bequest is common in our realistic world and is always regarded by the public as unequal inheritance. The bequests to charities will be gathered and then equally redistributed among the survivors in our model. We find that when all the decedents choose the second kind of bequest, the final distribution is the Gibbs exponential function. When all the decedents choose the first kind of bequest, the result is condensation that a single individual accumulates all the available wealth. When an increasing portion of decedents choose the one-heir bequests, the exponential distribution evolves towards a power law shape (accompanied by deteriorating inequality). This shape firstly appears from the intermediate range of wealth and extends towards the top end of the simulated distribution, while the distribution remains exponential for high values of the wealth. At the same time, the Gini coefficient increases and the wealth accumulation becomes serious. At last, we analyze the source of the inequality. We find that not only unequal inheritances, but also equal division of the charity's wealth can relatively contribute to an inequality of wealth distribution.

1. Introduction

The distribution of wealth and income among agents is always an important research in the field of economics. It has been found to have some robust and stable features independent of time and specific economic or social conditions. The century old Pareto law [1] in economics states that the upper tail of the distribution or income follows a power-law $f(x) \propto x^{-1-\gamma}$ with exponent γ robust in time. As for the middle and lower income range, distribution is believed to have a Gibrat's [2] or Gamma [3] form. The piecewise wealth distribution has been verified by a lot of empirical studies including personal income distribution in Japan [4], Italy [5], the United Kingdom and the United States [6]. The same feature has even been observed from the wealth distribution in an Egyptian society [7] and the distribution in the medieval Hungarian aristocratic society [8].

By analogy with a physical system which consists of a large number of simple dynamical elements having local interactions, asset exchange models, of simple rules and interesting results, were proposed trying to explain these empirical data. The model considers a conserved system that consists of N interacting agents. Each of the agents is endowed with a portion of wealth. At each time step, wealth transfers between two randomly selected agents according to some pre-specified deterministic or stochastic rules. A. Dragulescu and V.M. Yakovenko found that when the exchanged fraction is a fixed or

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random share of one interacting agent's wealth, the obtained wealth distribution falls into a Gibbs-exponential type [9]. Saving propensity was then introduced into the random exchange model and a series of distributions were obtained [10], which were later justified to have the Gamma form [11]. Another interesting model is the yard sale model [12] where two randomly selected agents interact according to a simple rule: the winner takes a share of the loser's wealth. A frequent result in these models is condensation, with one agent eventually acquiring all wealth. In order to avoid this situation, several methods have been proposed, including revising exchange rules so that poorer agents are favored [13,14] or introducing taxes and regulations [15]. The dynamics driving the system to condensed state and the entropy behavior when the system approaches condensations were also investigated [16].

However, there are still a lot of other essential dynamics apart from transactions in the realistic market world being neglected in the current asset exchange models. And one of them is the wealth transfer that happens at the end of an agent's life. In most cases, people will not use up all their wealth intentionally for the consideration of leaving bequests to others. There are evidences [17] that a significant fraction of total saving is motivated solely by this desire. The motive of giving intergenerational bequests is, by a leading theory [18], altruism—they care about the well-being of their children and grandchildren. Another kind of bequest is the charitable bequest. Charitable bequests are most often the results of a sincere desire to help others. Besides, this kind of bequest can also be a direct result of the high estate taxes [19]. According to a empirical study [20], approximately 2.4 million adults died in the United States in 1999, leaving estates valued at over $196 billion to their families, charities, federal and state governments (via estate taxes), or others. The top three bequest receivers are the surviving spouse, the federal government for estate taxes, state governments and charities.

In this paper we investigate the effects of different kinds of bequests on the final wealth distribution. Our model is based on a multi-agent dynamic system. There are no traditional transactions but only bequests happening among agents. We consider only two kinds of bequests: to a single agent and to charities. The former kind of bequests usually happens among family members. The heir could be the deceased person's surviving spouse, son and daughter, or other relatives. We just randomly picked up one agent from the survivors and assume it is exactly the heir in our model. The bequests to charities will then be gathered and equally redistributed among all the other survivors. We want to find out which shape of wealth distribution different combination of the two kinds of bequests will lead to.

2. Model

We study a conserved system that consists of N agents and W units of wealth. The time is a discrete variable ($t = 1, 2, \ldots, T$). Each agent is endowed with a fixed amount of wealth $w_0 = W/N$ at the first time step. At each time step, random β fraction of agents die and all their wealth is bequeathed. The α fraction of dead transmits all their wealth to a single heir separately while the full wealth of the other $1-\alpha$ fraction of the dead is redistributed equally among all the other survivors. The corresponding $\beta * \alpha * N$ heirs are randomly selected from the survivors. In order to guarantee enough heirs for the one-heir dead agents, we keep $\beta < 1/(\alpha + 1)$. In order to keep the number of agents constant, in every time step, the same number $\beta * N$ new agents with wealth 0 are incorporated. We put the agents in sets A, B, C, D, E and F. Table 1 shows the relationship and membership of each set.

In every time step, we make an arbitrary one-to-one mapping f from C to E.

$$f(i) = j (i \in C, j \in E). \tag{1}$$

$S(t)$ is the total wealth donated to charities at the time step t, $w(i, t)$ is the wealth of agent i at the time step t, then we get:

$$S(t) = \sum_{i \in D} w(i, t). \tag{2}$$

The wealth of agent i evolves according to the following rule:

$$w(i, t+1) \begin{cases} 0 & i \in A \\ w(i, t) + w(f^{-1}(i), t) + \dfrac{S(t)}{(N - \beta * N)} & i \in E \\ w(i, t) + \dfrac{S(t)}{(N - \beta * N)} & i \in F. \end{cases} \tag{3}$$

The gross wealth at time step $t + 1$ is

$$\begin{aligned} W(t+1) &= \sum_{i \in U} w(i, t+1) \\ &= \sum_{i \in E} \left[w(i, t) + w(f^{-1}(i), t) + \frac{S(t)}{N - \beta * N} \right] + \sum_{i \in F} \left[w(i, t) + \frac{S(t)}{N - \beta * N} \right] \\ &= \sum_{i \in B} w(i, t) + \sum_{i \in E} w(f^{-1}(i), t) + S(t) \\ &= \sum_{i \in B} w(i, t) + \sum_{j \in C} w(j, t) + \sum_{i \in D} w(i, t) = W(t). \end{aligned}$$

Therefore, wealth in this system is conserved.