Physica A 392 (2013) 1439–1451

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Efficiency and probabilistic properties of bridge volatility estimator

S. Lapinova [a], A. Saichev [b,*], M. Tarakanova [b]

[a] National Research University Higher School of Economics, Russia
[b] Nizhni Novgorod State University, Russia

ARTICLE INFO

Article history:
Received 24 March 2012
Received in revised form 9 November 2012
Available online 27 November 2012

Keywords:
Volatility estimators
Probabilistic properties
Efficiency
Unbiasedness

ABSTRACT

We discuss the efficiency of the quadratic bridge volatility estimator in comparison with Parkinson, Garman–Klass and Roger–Satchell estimators. It is shown in particular that point and interval estimations of volatility, resting on the bridge estimator, are considerably more efficient than analogous estimations, resting on the Parkinson, Garman–Klass and Roger–Satchell ones.

1. Introduction

Volatility, defined as the variance of the increments of the log-price over a specific time interval, is a universally used risk indicator. Most of the existing high-frequency variance estimators are modifications of the well-known realized volatility (see, for instance, Refs. [1–3]), and are based on the knowledge of the open and close prices of n time-step intervals dividing the whole time interval of interest.

With the growing availability of high-frequency tick-by-tick price time series, a number of new efficient volatility estimators have been developed (see, for instance, Refs. [4–6]). We present here a comparative analysis of the efficiency of the quadratic bridge volatility estimator [7] and the well-known Parkinson (PARK) [8], Garman–Klass (GK) [9] and Roger–Satchell (RS) [10,11] volatility estimators, based on high and low values of the log-price increments within given time intervals. Some fruitful information and detailed analysis of stochastic price models and efficiency of high–low-close volatility estimators one can find in the recent article Ref. [12]. Detailed and valuable discussion of stochastic volatility models and volatility estimators, related to the topic of the present paper, are provided in G. Ramey and V. Ramey Ref. [13] and in Bonanno et al. Refs. [14,15].

We show that the high–low quadratic bridge estimator, discussed in this work, is significantly more efficient, for the point and interval volatility estimations, than the above-mentioned PARK, GK and RS estimators, at least in the framework of the geometric Brownian motion with a drift model of the price stochastic process. Notice that some related results concerning statistical properties of volatility estimators were obtained in Saichev et al. Ref. [16], where they have discussed constructions of most efficient volatility estimators. It was shown that efficiencies of the pointed out most efficient estimators are very close to the efficiency of the quadratic bridge estimator discussed in this paper. From another side, the shortcoming of the most efficient estimators, discussed in Ref. [16], is that they have much more complicated structure than the quadratic bridge estimator, discussed in this paper.

For the Brownian motion model of log-price process, the advantage of the quadratic bridge estimator can be intuitively understood as follows. It is well-known that the high and low values of a Brownian motion process are most probably found

* Correspondence to: Bodenacherstrasse 67, 8121, Benglen, Switzerland. Tel.: +41 44577050541.
E-mail address: saichev@hotmail.com (A. Saichev).

0378-4371/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2012.11.047

in the neighborhood of the edges of the observation interval. In contrast, by construction of the bridge, its high and low values are in general distant from the edges (see Fig. B.1 at the end of the article, where are depicted distributions of instants of high value occurrence for the Brownian motion and the corresponding bridge). As a result, the high and low of a bridge incorporate significantly more information about the variability of the original stochastic process than its own high and low values.

It is worthwhile to stress additionally that one cannot calculate intraday volatility, due to the lack of historical financial markets data, during the middle (and even at the end) of the previous century. But now there are accessible even tick-by-tick price data for most financial markets and stocks. So, one can now easily calculate values of the intraday bridge high–low volatility estimators.

Notice in conclusion that it seems at first glance that high-frequency (or even tick-by-tick) realized-volatility estimators might be more efficient than any high–low estimators. It is, of course, right in the framework of the geometrical Brownian motion model of the price stochastic behavior. Unfortunately, the mentioned model failed at the small time scales. Moreover, there is a microstructure noise effect, making high-frequency realized-volatility estimators biased and less efficient even in the case of zero drift. Concerning quantitative description of the microstructure noise effect see our paper [17] and the references therein.

The paper is organized as follows. In Section 2 a short description of high–low volatility estimators, including the quadratic bridge estimator, suggested in this work, is given. In Section 3, the statistical description of high–low volatility estimators, in the framework of the Brownian motion model of the log-price stochastic process, is discussed in detail. In Section 4, we compare the efficiency of PARK and quadratic bridge estimators. In Section 5, we give a comparative probabilistic analysis of the interval estimations, resting on the bridge, PARK, GK and RS volatility estimators. In Section 6 the results of statistical testing of the above-mentioned volatility estimators are described. In Section 7 we draw the conclusions.

2. Examples of volatility estimators

Consider the dependence on time t of the price $P(t)$ of some financial instrument. As a rule, in discussing volatility, one consider its logarithm

$$X(t) := \ln P(t).$$

Let us point out one of the conventional volatility $V(T)$ definitions, which we are using in this work. It is the variance

$$V(T) := \mathbf{Var}[Y(t,T)] = \mathbf{E}[Y^2(t,T)] - \mathbf{E}^2[Y(t,T)]$$

of the log-price increment $Y(t,T) := X(t+T) - X(t)$ within a given time interval duration T.

Recall that GK [9], PARK [8] and RS [10] volatility estimators are resting on the high and low values:

$$H := \sup_{t' \in (0,T)} Y(t,t'), \qquad L := \inf_{t' \in (0,T)} Y(t,t').$$

Accordingly, the PARK estimator is equal to

$$\hat{V}_p := (H-L)^2 / \ln 16, \tag{1}$$

while the GK estimator is given by the expression

$$\begin{aligned} &\hat{V}_g := k_1(H-L)^2 - k_2(C(H+L) - 2HL) - k_3C^2, \\ &k_1 = 0.511, \qquad k_2 = 0.0109, \qquad k_3 = 0.383. \end{aligned} \tag{2}$$

Here $C := Y(t,T)$ is the close value of the log-price increment. Recall also the RS estimator, equal to

$$\hat{V}_r := H(H-C) + L(L-C).$$

Besides the mentioned well-known estimators, we discuss the quadratic bridge estimator. Below we call it briefly the *bridge estimator*. Before we define it, recall the definition of the bridge $Z(t,t')$ of stochastic process $Y(t,t')$. It is equal to

$$Z(t,t') := Y(t,t') - \frac{t'}{T}Y(t,T), \quad t' \in (0,T). \tag{3}$$

Let introduce the high and low of the bridge:

$$\mathcal{H} := \max_{t' \in (0,T)} Z(t,t'), \qquad \mathcal{L} := \min_{t' \in (0,T)} Z(t,t').$$

Accordingly, the bridge volatility estimator mentioned above is given by

$$\hat{V}_b := \kappa(\mathcal{H} - \mathcal{L})^2. \tag{4}$$

The value of the factor κ will be calculated later on.