Physica A 392 (2013) 1452–1458

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Multifractal detrended fluctuation analysis of the Chinese stock index futures market

Xinsheng Lu [a,*], Jie Tian [b], Ying Zhou [c], Zhihui Li [d]

[a] Department of Economics and Finance, SEM, Tongji University, Shanghai 200093, China
[b] Center for Resource Economics and Management, Northwest A&F University, Yangling 712100, China
[c] Department of Economics, Auckland University of Technology, Auckland, New Zealand
[d] School of Economics, Jinan University, Jinan, China

ARTICLE INFO

Article history:
Received 3 June 2012
Received in revised form 29 August 2012
Available online 26 November 2012

Keywords:
Multifractality
Stock index futures
MF-DFA
Generalized Hurst exponent

ABSTRACT

Based on the multifractal detrended fluctuation analysis (MF-DFA) and multifractal spectrum analysis, this paper empirically studies the multifractal properties of the Chinese stock index futures market. Using a total of 2942 ten-minute closing prices, we find that the Chinese stock index futures returns exhibit long-range correlations and multifractality, making the single-scale index insufficient to describe the futures price fluctuations. Further, by comparing the original time series with the transformed time series through shuffling procedure and phase randomization procedure, we show the existence of two different sources of the multifractality for the Chinese stock index futures market. Our results suggest that the multifractality is mainly due to long-range correlations, although the fat-tailed probability distributions also contribute to such multifractal behaviour.

1. Introduction

Since the launch of stock index futures on the China Financial Futures Exchange (CFFE) in April 2010, China's financial market has become more complete and integrated with the rest of world. The index futures are referenced to the China stock index 300 (CSI300), which consists of 300 RMB-denominated A shares actively traded on the Shanghai and Shenzhen stock exchanges.

This paper aims to empirically test whether returns on the newly established Chinese stock index futures exhibit long-range correlations and multifractal patterns. The term *fractal* was coined by Mandelbrot [1] to characterize a rough or fragmented geometric shape that displays a large degree of self similarities within its own fractional dimensions. In recent years, fractal patterns have been extensively studied in diverse fields, ranging from physics to economics and finance.

In the literature, several approaches have been developed and applied to the exploration of fractal properties. For instance, the rescaled adjusted range analysis (R/S analysis) was introduced by Hurst [2] for his hydrological study. The well-known Hurst exponent, which is directly related to the fractal dimension, has now been widely used in the analyses of financial market volatility. Due to the difficulty of R/S analysis in capturing long-range correlations of non-stationary series, Peng et al. [3] proposed the detrended fluctuation analysis (DFA) method in order to analyse DNA sequences. Although DFA has become a widely used approach for the determination of monofractal scaling properties, it cannot be applied to describe the multi-scale and fractal subsets of the time series. Based on a generalization of DFA, Kantelhardt et al. [4] advanced the multifractal detrended fluctuation analysis (MF-DFA) for the multifractal characterization of non-stationary time series.

* Corresponding author. Tel.: +86 15800368545.
E-mail addresses: normanxlu@tongji.edu.cn (X. Lu), Tianjie2009@163.com (J. Tian), Ying.Zhou@aut.ac.nz (Y. Zhou), lizhihui1982@gmail.com (Z. Li).

0378-4371/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2012.11.037

As a robust and powerful technique for the verification of multifractal behaviour, MF-DFA has so far been applied to various markets, including international crude oil markets [5], foreign exchange markets [6], stock markets [7], gold markets [8], and agricultural commodity futures markets [9]. Scholars have also studied the cross-correlations between two non-stationary time series [10–13] by generalizing DFA and MF-DFA analyses with an emphasis on detrended covariance. However, none of the existing studies has applied the MF-DFA approach to the stock index futures market, in particular in the context of Chinese economy.

Using 10-minute closing prices of the CSI300 futures contract, this paper enriches the MF-DFA literature by analysing the characteristics of China's stock index futures market. The purpose is two-fold: first, to demonstrate that returns to the Chinese stock index futures exhibit long-range correlations and multifractal patterns; second, to discuss the sources of such multifractality in the futures market. To our best knowledge, this study represents the first attempt to explore the multifractal properties of the Chinese stock index futures returns.

The research in the area of stock index futures has largely focused on well-developed markets [14–21], while emerging futures markets have not yet received much attention. In particular, very limited research was previously done on the emerging stock index futures market in China. Until quite recently, Wen et al. [22] examined the market efficiency and hedging effectiveness of the Chinese index futures market, and Yang et al. [23] investigated the intraday price discovery and volatility transmission between the Chinese CSI300 index market and the stock index futures market. Nonetheless, none of the prior studies has tackled the multifractality and its possible sources for the Chinese stock index futures market.

The remainder of the paper is organized as follows. Section 2 specifies the MF-DFA methodology used for this study. Section 3 describes our dataset. Section 4 presents the empirical results. Section 5 is reserved for the conclusion.

2. Methodology

Let $\{x_k, k = 1, \ldots, N\}$ be a time series, where N is the length of the series. The MF-DFA procedure consists of the following steps.

Step 1: Determine the profile

$$Y_i = \sum_{k=1}^{i} (x_k - \bar{x}), \quad i = 1, 2, \ldots, N \tag{1}$$

where $\bar{x}$, $\bar{x} = \frac{1}{N}\sum_{k=1}^{N} x_k$, denotes the averaging over the whole time series.

Step 2: Divide the profile Y_i into $N_s = \text{int}\left(\frac{N}{S}\right)$ non-overlapping segments according to the length of s. Since the length N is often not a multiple of the considered time scale s, the same dividing procedure is repeated starting from the opposite end in order not to disregard some part of the time series. Thereby, $2N_s$ segments are obtained altogether.

Step 3: Calculate the local trend for each sub-interval υ, where $\upsilon = 1, 2, \ldots, 2N_s$. Then we get the fitting equation

$$P_\upsilon(i) = a_0 + a_1 i + \cdots a_k i^k, \quad i = 1, 2, \ldots, s; k = 1, 2, \ldots \tag{2}$$

where $s \geq \max\{k + 2, 10\}$.

Here, $k = 1, 2, \ldots$ means polynomial with order k is used when making a regression of s time series points.[1]

Step 4: Determine the variance by eliminating the local trend of each sub-interval υ.

$$F^2(s, \upsilon) = \frac{1}{s}\sum_{i=1}^{s}\left[Y\left((\upsilon - 1)s + i\right) - P_\upsilon(j)\right]^2, \quad \upsilon = 1, 2, \ldots, N_s \tag{3}$$

or

$$F^2(s, \upsilon) = \frac{1}{s}\sum_{i=1}^{s}\left[Y\left(N - (\upsilon - N_s)s + i\right) - P_\upsilon(j)\right]^2, \quad \upsilon = N_s + 1, N_s + 2, \ldots, 2N_s. \tag{4}$$

Here, $P_\upsilon(j)$ is the fitting polynomial with order m in segment υ (conventionally, called mth order MF-DFA and wrote MF-DFAm).

Step 5: Average over all segments to obtain the qth order fluctuation function.

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{\upsilon=1}^{2N_s}\left[F^2(s, \upsilon)\right]^{\frac{q}{2}}\right\}^{\frac{1}{q}} \quad q \neq 0 \tag{5}$$

$$F_0(s) = \exp\left\{\frac{1}{4N_s}\sum_{\upsilon=1}^{2N_s}\ln\left[F^2(s, \upsilon)\right]\right\} \quad q = 0. \tag{6}$$

[1] For a detailed discussion of detrending method with varying polynomial orders, see Horvatic et al. [24].