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The emergence of scaling laws search dynamics in a particle swarm optimization

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ABSTRACT

This paper investigates the search dynamics of a fundamental particle swarm optimization (PSO) algorithm via gathering and analyzing the data of the search area during the optimization process. The PSO algorithm exhibits a distinct performance when optimizing different functions, which induces the emergence of different search dynamics during the optimization process. The simulation results show that the performance is tightly related to the search dynamics which results from the interaction between the PSO algorithm and the landscape of the solved problems. The Lévy type scaling laws search dynamics emerges from the process in which the PSO algorithm shows good performance, while the Brownian dynamics appears after the algorithm has stagnated due to the premature convergence. The Lévy dynamics characterized by a large number of intensive local searches punctuated by long-range transfers is an indicator of good performance, which allows the algorithm to achieve an efficient balance between exploration and exploitation so as to improve the search efficiency.

1. Introduction

The scaling laws strategy is widely applied by animals in nature when searching for food, which is referred to as Lévy flights, i.e., the search step lengths (l) of these predators follow a power-law distribution (the so-called Pareto–Lévy distribution): the density function $f(l) \propto l^{-\alpha}$, where α is the scaling exponent [1]. Quantities following power-law form are not well characterized by their typical or average values, and they have the scale invariance property, leading to the same distribution shape repeated across all scales. The flight lengths of many animals have been found to exhibit long-tailed power-law distributions (such as blue shark, mola, tuna, jackals, spider monkey, herbivores, soil amoebas, freshwater Hydra cells, mussels, etc.) [2–9]. Why Lévy flights as a foraging strategy are ubiquitous is still a secret to us, and some investigations think the Lévy motion is an efficient foraging method. The chance of returning to the same place in Lévy motion is less than in a normal diffusion process. Therefore, the Lévy motion especially optimizes the predator–prey encounter in a food scarce environment [10–12]. Meanwhile, the analysis on the data obtained by bank notes, mobile phones and GPS also showed the emergence of scaling laws in human activity [13–17]. The fat tail of jump size is thought to be the results of human decision processes with priority execution [17,18], which generates a strong heterogeneous distribution of waiting time and leads to the existence of scaling laws in human behavior. The main principle behind preferential choice is similar to 'preferential attachment' to generate a scale-free network [19]. Phase transitions and critical phenomena can also generate power-law distributions due to the divergence of some quantity [20]. The power-law distributions can also exist

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in the systems of self-organized criticality (SOC) [21]. However, the highly optimized tolerance (HOT) mechanism thinks the emergence of a power-law distribution is deliberately planted for optimization rather than self-organization [22]. In addition, some researchers believe the scaling laws of spatial networks can improve the performance of networked systems through optimizing the information entropy [23,24]. Therefore, understanding the emergence of scale laws is a crucial and challenging problem.

Since systems with scaling properties exhibit good performance, it is interesting to incorporate the power-law property into heuristic optimization algorithms. Previous investigations focused on the improvement of performance through introducing the power-law property into the algorithms [25–27]. However, the inherent searching dynamics of the algorithm is little addressed. More recently, the dynamic property of the optimization process of the genetic algorithm was investigated and the topology of the information flow network was found to own the scale-free property, which is probably due to the fitness preferential selection strategy [28]. Actually, the evolutionary dynamics reflects the key search pattern of an algorithm. Understanding the search pattern may provide direct insights of the optimization process, and furthermore offer new methods for the design of algorithms.

In this paper, we investigate the search dynamics of a fundamental particle swarm optimization (PSO) algorithm by gathering and studying a process parameter, referred to search area. Similar to the jump length of foraging movement, the statistical property of the search area data characterizes the search pattern of the algorithm in the optimization process. The statistical analysis on the search area data shows that the scaling laws search pattern can inherently emerge from highly efficient optimization processes. The emergence of scale laws also confirms the assumption that the scaling laws are relevant to the state of high efficiency. In contrast, the Lévy search dynamics is replaced by a Brownian type behavior after the algorithm has been trapped by local minima. The Brownian search is characterized by intensive local scans around the local minimum that leads to a vicious circle. The trapped algorithm results in a slowly diffusive search which in turn prevents the algorithm from escaping out of the local optimum.

The remainder of this paper is organized as follows. Section 2 introduces the PSO algorithm, the search area parameter, the benchmark functions, and the numerical experiment settings. Section 3 analyzes the statistical properties of the PSO evolution process, and gives the results. Section 4 summaries the paper.

2. Methods

2.1. Particle swarm optimization

Particle swarm optimization (PSO), inspired by the social behavior of bird flocking or fish schooling, is a population based computational technique [29,30]. It has been widely applied in solving the practical optimization problems [31–34] due to its effectiveness and simplicity in implementation. The PSO has a swarm of candidate solutions, here dubbed particles, who move around in the search-space according to the updating rules of each particle's position and velocity. The movement of each particle is influenced by both its best known local position and the best known position among its neighborhood, i.e., the population expects to move toward the best solutions through information exchange. Formally, let $f : R^D \rightarrow R$ be the fitness or cost function of individuals which should be minimized, where D is the dimension of the solution space. Let P be the number of particles in the population, where each particle i has a position $\boldsymbol{x}_i = [x_{i1}, \ldots, x_{id}, \ldots, x_{iD}] \in R^D$ in the solution space (here x_{id} is the d-th dimension axis of the particle i) and a velocity $\boldsymbol{v}_i = [v_{i1}, \ldots, v_{id}, \ldots, v_{iD}] \in R^D$. Let $\boldsymbol{pb}_i$ be the best known position of particle i and let $\boldsymbol{g}_i$ be the best known position among i's neighbors. Then a particle is moving in the solution space according to

$$v_{id} = \gamma(v_{id} + c_1 rand_1(pb_{id} - x_{id}) + c_2 rand_2(g_{id} - x_{id})), \quad (1)$$

$$x_{id} = x_{id} + v_{id}, \quad (2)$$

where c_1 and c_2 are the acceleration coefficients, and $rand_1$ and $rand_2$ are two uniformly distributed random numbers which are independently generated within [0, 1]. The constriction factor γ is used to avoid the unlimited growth of the particles' velocity, so that the PSO is also known as constricted PSO [31].

2.2. Search pattern parameters

The step length $L(t)$ measures the distance between two successive population centers ($x_{md}(t) = \frac{1}{D}\sum_{i=1}^{N} x_{id}(t)$) at the t-th iteration:

$$L(t) = \sqrt{\sum_{d=1}^{D} (x_{md}(t-1) - x_{md}(t))^2} \quad t = 2, \ldots, Mstep, \quad (3)$$

where $Mstep$ is the maximum number of steps to obtain a result.