Physica A 392 (2013) 1–6

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Minireview

Phase diagrams of the transverse Ising antiferromagnet in the presence of the longitudinal magnetic field

Minos A. Neto [a,*], J. Ricardo de Sousa [a,b]

[a] Departamento de Física, Universidade Federal do Amazonas, 3000, Japiim, 69077-000, Manaus-AM, Brazil

[b] National Institute of Science and Technology for Complex Systems, 3000, Japiim, 69077-000, Manaus-AM, Brazil

ARTICLE INFO

Article history:
Received 25 February 2012
Received in revised form 24 July 2012
Available online 14 August 2012

Keywords:
Effective-field theory
Transverse field
Ising antiferromagnet

ABSTRACT

In this paper we study the critical behavior of the two-dimensional antiferromagnetic Ising model in both uniform longitudinal (H) and transverse (Ω) magnetic fields. Using the effective-field theory (EFT) with correlation in single site clusters we calculate the phase diagrams in the $H-T$ and $\Omega-T$ planes for the square lattice. We have only found second order phase transitions for all values of fields and reentrant behavior was not observed.

Contents

1. Introduction

In the last decade there has been an increasing number of works dealing with magnetic models to study quantum phase transitions. In particular, considerable interest has been directed to the transverse Ising model (TIM) used to describe a variety of physical systems [1–3]. It was originally introduced by de Gennes [1] as a pseudospin model for hydrogen-bonded ferroelectrics such as KH_2PO_4 in the order–disorder phenomenon with tunneling effects. It has been successfully used to also study a number of problems of phase transitions associated with order–disorder phenomena in other systems [4,5]. It provides a good description to analyze some real anisotropic magnetic materials in a transverse field [6].

Theoretically, various methods have been employed to study the criticality of the TIM such as renormalization group (RG) [7], effective field theory (EFT) [8–10], mean field theory (MFA) [11], cluster variation method (CVM) [12], pair approximation (PA) [13], Monte Carlo (MC) simulations [14], and so on. The critical behavior of the one-dimensional TIM has already been established through exact results, where the ground-state energy, the elementary excitations and the correlation functions were obtained [15]. The TIM is among the simplest conceivable classes of quantum models in statistical mechanics to study quantum phase transitions [16,17]. The ferromagnetic TIM has been studied intensively [7–10,13,14]. The critical and thermal properties of the ferromagnetic and antiferromagnetic TIM are equivalent, but the properties of

* Corresponding author.
E-mail addresses: minos@pq.cnpq.br (M.A. Neto), jsousa@pq.cnpq.br (J.R. de Sousa).

0378-4371/$ – see front matter
doi:10.1016/j.physa.2012.07.073

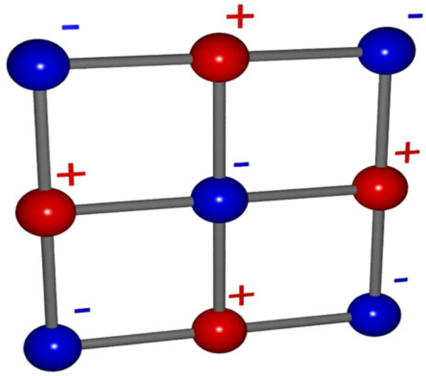

Fig. 1. Ground state of the quantum Ising antiferromagnet on an square lattice described by the Hamiltonian given in Eq. (1).

these two models are very different at not null longitudinal field ($H \neq 0$). For example, the ground-state phase transition in the ferromagnetic TIM is smeared out by the longitudinal field in contrast to the antiferromagnetic TIM for which the phase diagram remains qualitatively the same at $H \neq 0$.

Experimental investigations on the metamagnetic compounds such as $FeBr_2$ and $FeCl_2$ [18] and $Ni(NO_3)_2 2H_2O$ [19,20], under hydrostatic pressure, have been performed. For example, in the $FeBr_2$ compound a sharp peak was observed in magnetization measurements under a field inclined by 33° with the c axis (perpendicular to the plane) of the crystal. They concluded that the peak was affected by the ordering of the planar spin components. It is obvious that the field can be decomposed into the longitudinal and transverse components. The model is described by the Ising Hamiltonian to which is added a term which represents the effects of the transverse field part. Due to the non-commutativity of the operators in the Hamiltonian, deriving eigenvalues is a very difficult problem. Therefore, many theoretical methods have been used to investigate this system [21–25].

From a theoretical point of view it is known that the effect of the transverse field in the Ising model TIM is to destroy the long-range order of the system. Many approximate methods [7–14,21–30] have been used to study the critical properties of this quantum model. Some years ago, a simple and versatile scheme, denoted by the differential operator technique [31], was proposed and has been applied exhaustively to study a large variety of problems. In particular, this technique was used to treat the criticality of the TIM [32] obtaining satisfactory quantitative results in comparison with more sophisticated methods (for example, MC). This method is used in conjugation with a decoupling procedure which ignores all high-order spin correlations (EFT). The EFT included correlations through the use of the van der Waerden identity and provided results which are much superior than the ones coming from the MFA. The TIM was first studied by using EFT [32] for the case of spin $S = 1/2$, and generalized [26] for arbitrary spin-$S \geqslant 1/2$.

On the other hand, the critical and thermal properties of the transverse Ising antiferromagnet in the presence of a longitudinal magnetic field have been somewhat studied in the literature [33,34]. Using the classical approach MFA and the density-matrix renormalization-group method (DMRG) [34], the ground state phase diagram in the $(H - \Omega)$ plane was studied for a one-dimensional lattice. A critical line separates the antiferromagnetic (AF) phase with long-range order (LRO) from the paramagnetic (P) phase with uniform magnetization. The quantum critical point $\delta_c \equiv \left(\frac{\Omega}{J}\right)_c$ decreases as $h_c \equiv \left(\frac{H}{J}\right)_c$ increases, and is null at $h_c = 2.0$. The MFA approach does not give the correct qualitative description of the critical line [34]. Firstly, the quantum fluctuations shift the ground critical point $\delta_c = 1.0$ to $\delta_c = 2.0$ at $h = 0$ which underestimates the critical value. Second, the form of the critical line shows an incorrect behavior around the critical point $h_c = 2.0$.

By using the EFT approach, Neto and Ricardo de Sousa [35] have studied the ground-state phase diagram of this quantum model on two-dimensional (honeycomb ($z = 3$) and square ($z = 4$)) lattices, and discussed the possibility of the existence of a reentrant behavior around the $h_c = z$ critical value. The spin correlation effects are partially taken into account in EFT, while they are entirely neglected in MFA. The differences of results given by using EFT and MFA show that the spin correlation is important on the phase diagram. The ground-state phase diagram in the $(H - \Omega)$ plane is qualitatively similar to the results of Fig. 1 in Ref. [34], but the reentrant behavior found by MFA occurs near $h_c = z$.

In the present paper, using EFT we investigate the quantum phase transitions of the Ising antiferromagnet in both external longitudinal and transverse fields. This work is organized as follows. In Section 2 we outline the formalism and its application to the transverse Ising antiferromagnet in the presence of a longitudinal magnetic field; in Section 3 we discuss the results; and finally, in Section 4 we present our conclusions.

2. Model and formalism

The model studied in this work is the nearest-neighbor (nn) Ising antiferromagnet in a mixed transverse and longitudinal magnetic field divided into two equivalent interpenetrating sublattices A and B, that is described by the following