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On the behavior of suspension of drops on an inclined surface

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ABSTRACT

The flow of drops suspended on an inclined surface, are studied by numerical simulations at finite Reynolds numbers. The flow is driven by the acceleration due to gravity, and there is no pressure gradient in the flow direction. The effect of the Reynolds number, the Capillary number and density ratio on the distribution of drops and the fluctuation energy across the channel are investigated. It is found that drops tend to stay away from the channel floor, which is consistent with the behavior observed in the granular flow regime. Drops that are less deformable will stay further away from the channel floor. Also, drops appear at a larger distance from the floor as the Reynolds number increases. Simulations at large density ratios show that results are more compatible with computer simulations of granular flows. The behavior observed here resembles more the granular flow regime when the restitution coefficient is low.

1. Introduction

The flow of deformable drops suspended in another fluid has a wide variety of practical applications. Dynamics of deformable drops in a Poiseuille flow has been studied for several decades using analytical, numerical and experimental methods. Many of those were restricted to Stokes and potential flows.

The motion of liquid drops and cells through narrow channels and tubes has been a matter of interest for many years, and there has been considerable efforts in this area. From a practical point of view, the importance of this subject stems from many applications such as blood flow through blood vessels, the recovery of oil, the combustion of fuel sprays, and emulsion transport through industrial ducts.

The application of the present study is more to processing devices such as hoppers, chutes, mixers or other transfer equipment. Granular materials, however, segregate if they are subjected to flow or external agitation in the presence of a gravitational field. Many studies of granular flows have focused on gravity driven chute flows owing to its practical importance in granular transportation. Due to the simplicity of this type of flow, development and testing of new theories and equipment are encountered on this flow.

The migration of neutrally buoyant solid particles in pipe flow at finite Reynolds numbers was first observed by Segre and Silberberg [1,2]. Their experimental studies showed that a rigid sphere is subject to radial forces which tend to carry it to a certain equilibrium position at about 0.6 tube radii from the axis. Experimental studies by Karnis et al. [3,4] showed that drops suspended in pipe flow could also exhibit the Segre–Silberberg effect. While experiments of Goldsmith and Mason used mostly a single drop and dilute suspensions, Kowalewski [5] conducted experiments on concentrated suspension of drops and measured the concentration and velocity profiles of droplet suspensions flowing through a tube.

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Campbell and Brennen [6] performed a computer simulation for chute flows of granular materials. Their results consisted of the velocity distribution, density and granular temperature. They considered the effect of density and the shear rate on the granular temperature.

Theoretical studies were based on solution of the Navier–Stokes equations using perturbation methods. Ho and Leal [7] studied the inertia-induced lateral migration of a neutrally buoyant rigid sphere in a Newtonian fluid, and indicated that the equilibrium position is about 0.6 of the channel half-width from the centerline in Poiseuille flow. Richardson [8] considered the behavior of a two-dimensional inviscid bubble in Stokes flow and observed that when surface tension effects are large, the cross-section of the bubble is circular.

Several numerical methods have been used in the past to study the behavior of multiphase systems in the presence of solid boundaries. These numerical methods include volume-of-fluid, Lattice-Boltzmann, finite difference, finite element and boundary-integral methods. For example, Ref. [9] used a boundary-integral algorithm to study the motion of a three-dimensional drop between two parallel plates at a low Reynolds number. They found that when a deformable drop is initially placed off the centerline, the drop migrates towards the channel centerline. Janssen and Anderson [10] used this algorithm and modified the Green's function to account for the effect of the walls for non-unity viscosity ratios. Zhou and Pozrikidis [11] simulated the pressure-driven flow of a periodic suspension of drops, and showed that when the viscosity of the drop is assumed to be equal to that of the suspending fluid, the drop migrates towards the centerline of the channel. Coulliette and Pozrikidis [12] simulated the transient motion of a periodic file of three-dimensional drops in a cylindrical tube by the boundary integral method. It was found that when the Capillary number is sufficiently small, drops deform from their initial spherical shape as they migrate towards the centerline, and approach a steady shape after a preliminary stage of rapid deformation. Feng et al. [13,14] conducted a two-dimensional finite element simulation of the motion of a solid particle in Couette and Poiseuille flow at finite Reynolds numbers. They observed that a neutrally buoyant particle exhibits the Segre–Silberberg effect in Poiseuille flow. Mortazavi and Tryggvasson [15] used a finite-difference/front tracking method to simulate the motion of two- and three-dimensional drops suspended in a pressure-driven flow at finite Reynolds numbers. They observed that in the limit of small Reynolds numbers, the motion of the drop depends strongly on the viscosity ratio. At a higher Reynolds number, the drop moves to an equilibrium position about halfway between the centerline and the wall, or it undergoes oscillatory motion.

Bayareh and Mortazavi [16] performed a dynamic simulation of deformable drops in simple shear flow at finite Reynolds numbers. The flow was studied as a function of the Reynolds number and the Capillary number, and a shear thinning behavior was observed.

Nourbakhsh and Mortazavi [17] studied the motion of deformable drops in Poiseuille flow at non-zero Reynolds numbers. The density distribution of drops across the channel was studied as a function of the Reynolds number and the Capillary number. Also, the effective viscosity increased with the Reynolds number.

Makse et al. [18] studied the dynamic processes of granular flows experimentally. Using a high-speed video camera, they studied a rapid flow regime where the rolling grains size segregate.

Makse [19] studied the segregation of granular mixtures in two-dimensional silos using a set of coupled equations for surface flows of grains. He studied the thick flow regime, where the grains are segregated in the rolling phase.

Spontaneous stratification of granular mixtures has been reported by Cizeau et al. [20], where a mixture of grains of different sizes and shapes was poured in a quasi-two-dimensional heap. They studied this phenomenon using two different approaches. First, they introduced a cellular automaton model that illustrated clearly the physical mechanism. Second, they developed a continuum approach, based on coupled equations for surface flows of granular mixtures that allowed them to make quantitative predictions for relevant quantities.

Makse et al. [21] studied granular materials size segregation when exposed to external periodic perturbations such as vibrations.

Here, we study the motion of drops suspended on an inclined surface. The effect of the governing non-dimensional parameters on the behavior of a suspension is studied in detail.

2. Governing equations and numerical method

2.1. Governing equations

The flow of a suspension of drops in another fluid at non-zero Reynolds numbers is governed by the Navier–Stokes equations. The Navier–Stokes equations are written in conservative form with variable physical properties. The surface tension is added to the formulation by a delta function that acts at the interface:

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla\cdot(\rho\mathbf{u}\mathbf{u}) = -\nabla P + \nabla\cdot\mu(\nabla\mathbf{u} + \nabla\mathbf{u}^T) - \int(\sigma\kappa\mathbf{n})\delta(x - X(s,t))\mathrm{d}s. \quad (1)$$

Hence, the equations are valid for both the drop and the ambient fluid. Here, $\mathbf{u}$ is the velocity field, p is the pressure, ρ is the density, μ is the viscosity, σ is the interfacial tension, κ is the curvature for two-dimensional flows, $\mathbf{n}$ is an outward unit normal to the drop surface and δ is a two-dimensional delta function. $\mathbf{x}$ is the Eulerian coordinate system; $\mathbf{X}$ is a Lagrangian