Physica A 392 (2013) 89–102

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Real-time fractal signal processing in the time domain

András Hartmann [a], Péter Mukli [a,1], Zoltán Nagy [a,1], László Kocsis [a], Péter Hermán [a,b], András Eke [a,*]

[a] Institute of Human Physiology and Clinical Experimental Research, Faculty of Medicine, Semmelweis University, Budapest, Hungary

[b] Department of Diagnostic Radiology, Yale University, New Haven, CT, USA

ARTICLE INFO

Article history:
Received 29 January 2010
Received in revised form 8 June 2012
Available online 11 August 2012

Keywords:
Fractal
Time series
Real-time analysis
DFA
SSC
Software

ABSTRACT

Fractal analysis has proven useful for the quantitative characterization of complex time series by scale-free statistical measures in various applications. The analysis has commonly been done offline with the signal being resident in memory in full length, and the processing carried out in several distinct passes. However, in many relevant applications, such as monitoring or forecasting, algorithms are needed to capture changes in the fractal measure real-time. Here we introduce real-time variants of the Detrended Fluctuation Analysis (DFA) and the closely related Signal Summation Conversion (SSC) methods, which are suitable to estimate the fractal exponent in one pass. Compared to offline algorithms, the precision is the same, the memory requirement is significantly lower, and the execution time depends on the same factors but with different rates. Our tests show that dynamic changes in the fractal parameter can be efficiently detected. We demonstrate the applicability of our real-time methods on signals of cerebral hemodynamics acquired during open-heart surgery.

1. Introduction

Ever since Mandelbrot and Van Ness [1] introduced the dichotomous model of fractional Gaussian noise and fractional Brownian motion (fGn and fBm, respectively) and later Mandelbrot [2] powerfully demonstrated the ubiquitous presence of scale-free structures and processes in nature, the concept of fractal analysis and the interpretation of this aspect of characterizing systems' complexity has advanced considerably. The idea of fractal analysis proved to be useful in the analyses of physical [3–5], computer networking [6,7], geological [8,9], economical [10,11], psychological [12,13], physiological [14–16] time series data, and in clinically relevant applications too [17–19].

Fractal time series analysis in the time domain has typically been done offline. In these implementations the signal has to remain available in full length and processed in multiple passes to yield a statistical measure for each and every scale. This strategy, however, becomes an obstacle in real-time applications, such as monitoring or forecasting, where the purpose of the application of fractal time series analysis should be a prompt assessment of a condition when the local fractal descriptor of temporal complexity changes due to concomitant alterations in the underlying system dynamics.

Owing to its independence from signal class and its apparent robustness [20–23], the DFA method has become a frequently used tool for time domain fractal analysis. Its unique advantage is that its precision is moderately affected by signal length [21], therefore it can be used on short time series [23,24]. DFA is closely related to the Scaled Windowed

* Correspondence to: Institute of Human Physiology and Clinical Experimental Research, Faculty of Medicine, Semmelweis University, Tűzoltó Street 37-47, P.O.B. 448, Budapest, 1446, Hungary. Tel.: +36 20 462 9063; fax: +36 13343162.

E-mail address: eke.andras@med.semmelweis-univ.hu (A. Eke).

1 Equally contributing authors.

0378-4371/$ – see front matter © 2012 Published by Elsevier B.V.
doi:10.1016/j.physa.2012.08.002

Nomenclature

Symbols and definitions

DFA	Detrended Fluctuation Analysis, DFA for short
SSC	Signal Summation Conversion method, SSC for short
ldDFA	Line-detrended Detrended Fluctuation Analysis
bdSSC	Bridge-detrended Signal Summation Conversion method
N	Length of time series
X_i	Time series (signal for short), where $i = 1, \ldots, N$
Y_i	Time series (obtained by cumulative summation of signal X_i), where $i = 1, \ldots, N$
n	Window size of detrending
$X'_{i,n}$	Detrended time series (detrended signal for short)
W	Number of different window sizes used
M	Sliding window size for real-time methods, $M \leq N$, which thus defines the largest time scale of the fractal analysis
fGn	Fractional Gaussian noise, a stationary signal with fractal autocorrelation structure
fBm	Fractional Brownian motion, a non-stationary signal with self-similar (fractal) structuring
H	Hurst exponent
H_{true}	The value of the Hurst exponent at which the time series was generated
$\hat{H}$	Estimated value of the Hurst exponent
$\hat{H}_{SSC}$	Estimated value of the Hurst exponent using SSC method, where for fGn $0 < \hat{H}_{SSC} < 1$, for fBm $1 < \hat{H}_{SSC} < 2$. This will be referred to as "extended", which allows a direct comparison to α of DFA.
α	Estimated value of the fractal parameter using DFA method, for an fGn signal $\alpha = \hat{H}$ and for an fBm signal $\alpha = \hat{H} + 1$

Variance (SWV) method introduced by Mandelbrot and van Ness [1] from which Eke et al. [15] designed the SSC method specifically for a reliable signal classification based on the dichotomy of the fGn/fBm model of Mandelbrot and Van Ness [1] which implies improved performance in estimating H too (see Fig.16 in the reference) [21]. There are other candidates – similar to DFA – to consider like the DMA (Detrended Moving Average) algorithm [25]. Several modifications of DFA have been developed in various fields [26–30] for detecting temporal variations in fractal descriptors. However these implementations cannot be regarded as optimized because one data point is dealt with in multiple passes, significantly increasing computational cost. Changing complexity – reflected in a fractal measure – of time series showing rapid dynamics can be quantified in real-time with an efficient algorithm. A promising attempt in the direction of using an efficient algorithm is the application of incremental discrete wavelet transform for multifractals reported by Brodu [31].

Therefore, the aims of this study were: (i) to develop software tools optimized for real-time analysis of fractality in the time domain (ii) to evaluate their performance in numerical experiments regarding precision, numerical stability, execution time, dynamic behavior, and (iii) to demonstrate their applicability. First, we briefly overview the offline algorithms of DFA [32] and SSC [15] methods, then introduce their real-time algorithms followed by the description of our strategy of performance analysis along with results of numerical experiments, and that of characterization of precision and limitations of the methods. Applicability is demonstrated by analyzing a time series recorded in a medical environment, where fast processing is a basic requirement.

2. Methods

2.1. Offline analysis

DFA and SSC have a similar algorithmic design in that both yield an estimate of the scaling exponent by relying on similar statistical descriptors. The block-diagram of the original algorithms [15,32] is shown in the left panel of Fig. 1.

The fact that the time series must be resident in full length while the loop for different window sizes is executed renders this type of analysis offline.

2.1.1. Detrending

Detrending removes a fitted polynomial trend from the sample [22,33,34]. Here we focus on the most common first order (linear) method of detrending:

$$f_i = m \times i + b$$
$$X_i = X'_i - f_i \quad (1)$$