Physica A 392 (2013) 184–193

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

# Measuring capital market efficiency: Global and local correlations structure

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A R T I C L E   I N F O

*Article history:*
Received 23 April 2012
Received in revised form 12 July 2012
Available online 11 August 2012

*Keywords:*
Capital market efficiency
Long-range dependence
Short-range dependence
Fractal dimension

A B S T R A C T

We introduce a new measure for capital market efficiency. The measure takes into consideration the correlation structure of the returns (long-term and short-term memory) and local herding behavior (fractal dimension). The efficiency measure is taken as a distance from an ideal efficient market situation. The proposed methodology is applied to a portfolio of 41 stock indices. We find that the Japanese NIKKEI is the most efficient market. From a geographical point of view, the more efficient markets are dominated by the European stock indices and the less efficient markets cover mainly Latin America, Asia and Oceania. The inefficiency is mainly driven by a local herding, i.e. a low fractal dimension.


## 1. Introduction

A concept of capital market efficiency is a central notion in financial markets theory [1,2]. This notion is generally used for an ideal image of the capital market enabling us to process relevant information to the fundamental price generation. If the relevant information to the fundamental price generation is completely processed by the capital market price mechanism, then such a capital market is said to be efficient. Thus the capital market efficiency accentuates the informational efficiency of capital markets. A notion of the efficient capital market represents such a capital market where prices on traded securities, e.g. stocks, bonds, or property, already reflect all available information and that investors are completely rational. Consequently, the notion of the efficient capital market represents a fair game pattern. No investor can have an advantage in predicting a return on an asset price, since no one has access to information not already available to everyone. It means that investors in the efficient capital market cannot expect to achieve abnormal returns systematically. In other words, the capital market is efficient if the fluctuations of returns are unpredictable [1–3].

Paradoxically, an achievement of the ideal efficient capital market, enabling efficient allocation of investments, brings about no activity of investors and no activity of speculators. Because real life experiences with capital markets have shown that there are investors who indeed have been beating the capital markets in the long-term, discrepancies from the above mentioned ideal state are existent and thus worth analyzing.

Testing the efficiency of various capital markets in different regions is a popular topic in financial journals (e.g. [4–8]). However, the hypothesis of market efficiency is standardly either rejected or not and markets are ranked quite infrequently. Moreover, the researchers majorly focus on a single method and comment on the results. And even more, the whole idea of testing or measuring capital market efficiency has been dealing with the joint-hypothesis problem (i.e. when we reject the efficiency of a specific market, it might be caused by a wrong assumption of the market's behavior) since its beginnings. This issue was also touched on by Fama himself [2]. In this paper, we try to bypass the problem by defining the efficient market as

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0378-4371/$ – see front matter 
doi:10.1016/j.physa.2012.08.003

a martingale. We then analyze the fractal dimension, and long-range and short-range dependence to describe and measure the efficiency of specific markets.

The Hurst exponent and a presence of long-term memory have been widely analyzed in recent years—in stock indices [9,10], interest rates [11], bonds [12], exchange rates [13] and others. The results vary depending on asset type and on geographical situation as well. Statistically significant long-range dependence was detected in some individual NYSE-listed stocks [14]. Even though the series of developed markets usually possess only short or no memory, emerging markets exhibit a different behavior [15,16]. Looking at a different frequency, a significant long memory was found for weekly returns of a large number of Greek stocks [17]. Cajueiro and Tabak [18] rank the markets according to their efficiency and suggest that the Hong Kong stock exchange is the most efficient one followed by Chinese A type shares and Singapore, and finally by Chinese B type shares, which indicates that liquidity and capital restrictions should be taken into consideration in efficiency testing and mainly interpretation.

We use the Hurst exponent $H$ and the fractal dimension $D$ to construct a new measure of market efficiency based on a deviation from the ideal state (the efficient market) from both local and global perspectives. If the results based on different measures vary, we can further distinguish between local (herding) and global (structure of correlations) effects. We use the fact that the measures are bounded and thus can be used to construct an informative norm representing the said deviation from the ideal state. The measure is estimated for 41 stock indices at different stages of development from the beginning of 2000 till the end of August 2011, i.e. the data set includes the DotCom bubble and its bursting as well as the current Global Financial Crisis.

The paper is structured as follows. In Section 2, we define the efficient capital market. Section 3 describes relationships between efficiency and the measures we use. In Section 4, we describe the methods used for the fractal dimension and Hurst exponent estimation. Section 5 covers the results and discusses the implications. Section 6 concludes. The main value of this paper lies in the fact that the proposed methodology bypasses the standard caveats of efficiency testing by building on the martingale definition of efficiency, using different methods and merging them into the efficiency measure. Such a rather bold path leads to very interesting and also meaningful results.

## 2. Capital market efficiency

We use a triple $(\Omega, \mathscr{T}, P)$ for expressing a probability space and the expression $\mathbb{E}[X|\mathscr{T}]$ for the conditional expectations. Let $\{\omega \in \Omega\}$ be a set of elementary market situations. Let $\mathscr{T}$ be some $\sigma$-algebra of the subsets of $\Omega$, $P$ is a probability measure on $\mathscr{T}$ and $\Omega$ is an information set. This structure gives us all the machinery for static situations involving randomness. For dynamic situations, involving randomness over time, a sequence of $\sigma$-algebras $\{\mathscr{T}_t, t \geq 0\}$ needs to be taken into consideration. Inclusion $\mathscr{T}_t \subset \mathscr{T}_{t+1}$ for all $t$ represents the information arriving in time $t$. Suppose all $\sigma$-algebras to be complete. Thus $\mathscr{T}_0$ represents initial information. On the other hand, a situation that all is known is represented by the expression $\mathscr{T}_\infty = \lim_{t\to\infty} \mathscr{T}_t$. Such a family $\{\mathscr{T}_{t\geq 0}\}$ is called a filtration; a probability space endowed with such a filtration, $(\Omega, \mathscr{T}, \{\mathscr{T}_t\}, P)$, is also called a stochastic basis.

Let $C = (\Omega, \mathscr{T}, \{\mathscr{T}_t\}, P)$ be a capital market with distinguished flows $\{\mathscr{T}_{t\geq 0}\}$ of $\sigma$-algebras filtered probability space. We also call $\{\mathscr{T}_{t\geq 0}\}$ an information flow, and an expression $\{S_t\}_{t\geq 0} \in M$ is a security price process. The efficient market is then defined as follows:

*A capital market $C = (\Omega, \mathscr{T}, \{\mathscr{T}_t\}, P)$ is called efficient if there exists P such that each security price sequence $S = \{S_t\}_{t\geq 0}$ is a P-martingale, i.e. the variables are $\mathscr{T}_t$-measurable and*

$$\mathbb{E}_P[|S_t|] < \infty, \qquad \mathbb{E}_P[S_{t+1}|\mathscr{T}_t] = S_t, \quad t \geq 0. \tag{1}$$

If a sequence $\{\xi_t\}_{t\geq 1}$ is the sequence of independent random variables such that $\mathbb{E}_P[|\xi_t|] < \infty$, $\mathbb{E}_P[\xi_t] = 0$ for $t \geq 1$, $\mathscr{T}_t^\xi = \sigma(\xi_1, \ldots, \xi_t)$, $\mathscr{T}_0^\xi = \{\emptyset, \Omega\}$, and $\mathscr{T}_t^\xi \subseteq \mathscr{T}_t$ then, evidently, the security price sequence $S = \{S_t\}_{t\geq 0}$, where $S_t = \xi_1 + \cdots + \xi_t$ for $t \geq 1$ and $S_0 = 0$, is a martingale with respect to $\mathscr{T}^\xi = \{\mathscr{T}_t^\xi\}_{t\geq 0}$, and

$$\mathbb{E}_P\left[S_{t+1}|\mathscr{T}_t\right] = S_t + \mathbb{E}_P\left[\xi_{t+1}|\mathscr{T}_t\right]. \tag{2}$$

If a sequence $S = \{S_t\}_{t\geq 0}$ is a martingale with respect to the filtration $\{\mathscr{T}_{t\geq 0}\}$ and $S_t = \xi_1 + \cdots + \xi_t$ for $t \geq 1$ with $S_0 = 0$, then $\{\xi_t\}_{t\geq 1}$ is a martingale difference, i.e. $\xi_t$ is $\mathscr{T}_t$-measurable, $\mathbb{E}_P\left[|\xi_t|\right] < \infty$ and $\mathbb{E}_P\left[\xi_t|\mathscr{T}_{t-1}\right] = 0$.

Thus in words, the capital market efficiency is represented by the martingale property of the security price processes. Note that this feature is primarily connected to uncorrelated returns of the price series.[1] Compared to the random-walk-based efficiency, the martingale is more general and does not assume the series to be locally stationary (homoskedastic), which would be quite unrealistic for the financial time series. Nevertheless, the martingale assumption gives enough information about the expected Hurst exponent and fractal dimension. Note that our information set $\Omega$ contains only the prices of the analyzed indices so that we test and measure the weak form of the capital market efficiency.

[1] Note that security price process $S = \{S_t\}_{t\geq 0}$ can be taken either as a simple price or a logarithmic price process. In our application, we use the more standard approach, i.e. the logarithmic prices.