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Territorial developments based on graffiti: A statistical mechanics approach

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ABSTRACT

We study the well-known sociological phenomenon of gang aggregation and territory formation through an interacting agent system defined on a lattice. We introduce a two-gang Hamiltonian model where agents have red or blue affiliation but are otherwise indistinguishable. In this model, all interactions are indirect and occur only via graffiti markings, on-site as well as on nearest neighbor locations. We also allow for gang proliferation and graffiti suppression. Within the context of this model, we show that gang clustering and territory formation may arise under specific parameter choices and that a phase transition may occur between well-mixed, possibly dilute configurations and well separated, clustered ones. Using methods from statistical mechanics, we study the phase transition between these two qualitatively different scenarios. In the mean-fields rendition of this model, we identify parameter regimes where the transition is first or second order. In all cases, we have found that the transitions are a consequence solely of the gang to graffiti couplings, implying that direct gang to gang interactions are not strictly necessary for gang territory formation; in particular, graffiti may be the sole driving force behind gang clustering. We further discuss possible sociological—as well as ecological—ramifications of our results.

1. Introduction

Lattice models have been extensively used in the physical sciences over the past decades to describe a wide variety of condensed matter equilibrium and non equilibrium phenomena, see e.g., the reviews in Ref. [1–3]. Magnetization was the original application, but the list has grown to include structural transitions in DNA [4–6], polymer coiling [7,8], cellular automata [9,10], and gene regulation [11–13] to name a few. The resulting models are certainly simplified, but what they lack in detail is compensated by their amenability to analytical and computational treatment—and, occasionally, to exact solution. Moreover, at least for the behavior in the vicinity of a continuous transition, the simplifications inherent in these approximate models may be presumed to be inconsequential. In short, lattice models have proved extremely useful in the context of the physical, biological and even chemical sciences. In more recent years, lattice models have also been applied to study social phenomena [14–16], such as racial segregation [17,18], voter preferences [19–21], opinion formation in financial markets [22–24], and language changes in society [25–27], offering insight into socioeconomic dynamics and equilibria. In this paper we consider the problem of gang aggregation via graffiti in what is—to the best of our knowledge—the first application of lattice model results to the emergence of gang territoriality.

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Scratching words or painting images on visible surfaces is certainly not a new phenomenon. Wall scribblings have survived from ancient times and have been used to reconstruct historical events and to understand societal attitudes and values. Today, graffiti (from the Italian *graffiare*, to scratch) is a pervasive characteristic of all metropolitan areas [28]. Several types of graffiti exist. Some are political in nature, expressing activist views against the current establishment; others are expressive or offensive manifestations on love, sex or race. At times, the graffiti is a mark of one's passage through a certain area, with prestige being attributed to the most prolific or creative tagger or to one who is able to reach inaccessible locations. The mark can be anything from a simple signature to a more elaborate decorative aerosol painting [29,30]. All of these types of graffiti are usually scattered around the urban landscape and do not appear to follow any predetermined spatio-temporal pattern of evolution. They affect the quality of life simply as random defacement of property, although sometimes they are considered art [31].

On the other hand, *gang* graffiti represents a much more serious threat to the public, since it is usually a sign of the presence of criminal gangs engaged in illegal or underground activities such as drug trafficking or extortion [32,33]. Street gangs are extremely territorial, and aim to preserve economic interests and spheres of influence within the neighborhoods they control. A gang's "turf" is usually marked in a characteristic style, recognizable to members and antagonists [34,35] with incursions by enemies often resulting in violent acts. The established boundaries between different gang factions are sometimes respected peacefully, but more often become contested locations where it is not uncommon for murders and assaults to occur [36]. It is here, on the boundaries between gang turfs, that the most intense graffiti activity is usually concentrated.

Several criminological and geographical studies have been presented connecting gang graffiti and territoriality in American cities [31,30,35]. In particular, it is now considered well-established that the spatial extent of a gang's area of influence is strongly correlated to the spatial extent of that particular gang's graffiti style or language. Furthermore, it is known that the incidence of gang graffiti may change in time, reflecting specific occurrences or neighborhood changes. For example, rival gangs may alternate between periods of truce and hostility, the latter being triggered by arrests or shootings. Similarly, boundaries may shift locations when the racial or socio-economic makeup of a neighborhood changes, creating new tensions, or when gang members migrate to new communities [28]. In all these cases, periods of more intense gang hostility are usually accompanied by intense graffiti marking and erasing by rival factions in contested or newly settled boundary zones [35].

The purpose of this paper is to present a mathematical model that includes relevant sociological and geographical information relating gang graffiti to gang activity. In particular, we study the segregation of individuals into well defined gang clusters as driven by gang graffiti, and the creation of boundaries between rival gangs. We use a spin system akin to a 2D lattice Ising model to formulate our problem through the language of statistical mechanics. In this context, the site variables s_i have two constituents which represent 'gang' and 'graffiti' types, respectively, and *phase separation* is assumed to be the proxy for gang clustering. For the purpose of simplicity, we consider only two gangs, hereafter referred to as the red and blue gang, whose members we refer to as *agents*. Lattice sites may be occupied by agents of either color or be void. Since gang members are assumed to tag their territory with graffiti of their same color, we also assign a graffiti index to each site representing the preponderance of red or blue markings.

In particular, agents are attracted to sites with graffiti of their same color, and avoid locations marked by their opponents. We deliberately avoid including direct interactions between gang members, so that "ferromagnetic" type gang–gang attractions exist only insofar as they are mediated by the graffiti. On one hand this is mathematically interesting: in the broader context of physical systems, interactions are often mediated but rarely are indirect interactions the subject of mathematical analysis. On the other hand, by excluding direct gang interactions, we can specifically focus on the role of graffiti in gang dynamics and segregation. Furthermore, as will be later discussed, under certain conditions, gang–gang couplings may be unimportant, and one of the primary conclusions of this work is that they appear to be unnecessary to account for the observed phenomena of gang segregation. In any case, we informally state without proof that all the results of this work also hold if explicit agent–agent interactions are included.

We thus write $s_i = (\eta_i, g_i)$, representing the agents and graffiti configuration at site i, respectively. The former component η_i is discrete allowing, for simplicity, at most one agent on each site. The latter g_i is continuous and, in principle, unbounded. We let $\mathbf{s}$ denote a spin configuration on the entire lattice, and in Section 2, propose a Hamiltonian, $\mathscr{H}(\mathbf{s})$, to embody all relevant sociological information. Once $\mathscr{H}(\mathbf{s})$ has been determined, the probability for the occurrence of a spin configuration $\mathbf{s}$ on a finite connected lattice $\Lambda \subset \mathbb{Z}^2$ is determined by the corresponding Gibbs distribution $\mathbb{F}(\mathbf{s})$. Note that due to the choices made on the range of the η_i, g_i values, $\mathbb{F}(\mathbf{s})$ is discrete in the η variables and continuous in the g ones. It is given by

$$\mathbb{F}(\mathbf{s}) = \frac{1}{\mathcal{Z}} \exp(-\mathscr{H}(\mathbf{s})),$$

where $\mathcal{Z}$ is the partition function for the finite lattice Λ formally provided by the expression

$$\mathcal{Z} = \sum_{\mathbf{s}\in\mathbb{S}} \exp(-\mathscr{H}(\mathbf{s})).$$

Here, $\mathbb{S}$ denotes the set of all possible configurations on Λ and the summation symbol is understood to be a summation over the discrete components and an integration over the continuous ones. As usual, we begin with a finite lattice and its associated boundary conditions, and obtain infinite volume results by taking the appropriate limits. Using techniques from