Physica A 392 (2013) 287–293

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Measuring persistence in a stationary time series using the complex network theory

Somaye Karimi, Amir H. Darooneh *

Department of Physics, Zanjan University, P.O.Box 45196-313, Zanjan, Iran

ARTICLE INFO

Article history:
Received 16 June 2012
Received in revised form 23 July 2012
Available online 13 August 2012

Keywords:
Complex network
Fractional Brownian motion
Determined fluctuation analysis

ABSTRACT

A growing interest exists currently in the analysis of time series by the complex network theory. Here we present a simple and quick way for mapping time series to complex networks. Using a simple rule allows us to transform time series into a textual sequence then we divide it into words with fixed size. Distinct words are nodes of the network, and we have complete control on the network scale by adjusting the word size. Two nodes are linked if their associated words co-occur in sequence. We show that the network topological measures quantify the persistence and the long range correlations in fractional Brownian processes. For a particular word size we assume some relations between the topological measures and the Hurst exponent which characterised the persistence in fractional Brownian processes.

1. Introduction

Time series is an emergent output of a system and contains much information about its microscopic details. By analysing the time series we aim to find the underlying generic features of the system and quantifying its information content.

Scaling analysis of time series detects the self-similar properties that are independent of the system's details. Self-similarity of complex temporal processes may be seen as scale invariance in the probability density function and the auto-correlation function. The auto-correlation function of a time series exhibits the mutual relationship between values of the process at different points in time against their time difference. In complex processes the auto-correlation asymptotically behaves as a power law that is characterised by an exponent. Several ways exist to measure such an exponent for a time series such as the spectral analysis, determined fluctuation analysis, Zipf analysis [1] and so on.

Recently the complex network theory appears as an useful method for understanding the complexity in systems. There are some attempts to use the concept of complex networks for studying the time series.

Yang and Yang have proposed a procedure for constructing complex networks from the correlation matrix of a time series [2]. In their method, all segments of a time series with length L are considered as nodes of the networks and two nodes are linked to each other if their correlation coefficient will be greater than a critical value. They found that the corresponding network for stock price time series has a scale free property but degree distribution of the returns network is well fitted with a Gaussian function.

Another strategy for converting a time series into a network is the visibility algorithm [3]. Each node in the network represents a point in series data; two arbitrary data values (t_a, x_a) and (t_b, x_b) are linked if the line segment connecting these points does not cross the time series at any other data that are placed between them. Several properties of time series

* Corresponding author. Tel.: +98 2415152545.
E-mail addresses: som.karimi@gmail.com (S. Karimi), darooneh@gmail.com, darooneh@znu.ac.ir (A.H. Darooneh).

0378-4371/$ – see front matter
doi:10.1016/j.physa.2012.07.077

can be inherited in the structure of the corresponding network by the visibility algorithm, accordingly the periodic, random and fractal series are mapped to the regular, random and scale free networks respectively.

We can transform a given time series which is continuous in value into a symbolic sequence by discretising its data values. All possible sub-sequences with length L play the role of the network nodes. Two sub-sequences are linked if they appear in the main sequence consecutively. Li et al. have presented such a scheme for construction of the multi-scale state space network to investigate the conformational fluctuation dynamics probed by single-molecule electron transfer, detected on a photon-by-photon basis [4].

Data values can be discretised into predetermined bins. These bins play the role of nodes in network. The nodes i and j are connected if as the time increases the value of bin i changes to the value of bin j in one step. Shirazi et al. quantified the effect of the long range correlations in several empirical time series by comparing the network topological measures of the original series with the shuffled one [5].

Zhang and Small introduced a method to construct a complex network from pseudoperiodic time series. In this method each node in a network is associated with a cycle in time series. Two nodes are supposed to be connected if their distance in phase space will be less than a predetermined value [6]. They found that the topological measures of the network represent a dynamics regime in time series. For example random networks correspond to noisy periodic signals and networks built from chaotic time series exhibit small world and scale free properties.

Using the time delay method we can embed a time series in phase space. Each point in phase space represents a node and two successive points are connected to form a network from a given time series. Distribution of motifs allows one to distinguish between different types of continuous dynamics such as: periodic, periodic with noise and chaotic time series [7].

A recurrence plot is a graphical tool specially suited to explore temporal patterns in time series. A time series of a single observable variable can be segmented into data blocks with size M. Each block represents a vector in M-dimensional space and also a node in the network. We can link two nodes if their distance in space is less than a threshold value [8]. By using the logistic map, Marwan et al. showed the potential of the complex network theory for detection of dynamical transitions. This method was developed and reviewed by some people [9,10].

In this work we suggest a simple and quick way for mapping time series into complex networks. First we transform the series into a textual sequence then divide it into fixed length words. Each word represents a node in the network and the corresponding nodes of two adjacent words in the text are linked. We quantify the persistence in time series that are sampled from one dimensional fractional Brownian motion processes by considering topological measures of their constructed networks.

We organise our paper as follows, in the next section we explain our method for transforming time series into complex networks, then we report our results in the third section. The effect of word size and series length are considered in the fourth section. Finally we summarise our work in the last section.

2. From time series to complex network

A time series is a series of values that are sampled at successive times from a temporal process; $\{x(t), \ldots, x(t+i\Delta), \ldots, x(t+N\Delta)\}$. The correlation of a time series with its own past and future values, is referred to as auto-correlation. The auto-correlation function is defined as,

$$C(\tau) \sim \langle x(t')x(t'+\tau)\rangle \tag{1}$$

where $\langle\cdots\rangle$ means averaging over all data values.

The power law form for the auto-correlation function, $C(\tau) \sim \tau^{-\alpha}$, discloses a pattern in time series, in a probabilistic sense. If some pattern exists we will be able to forecast how the values of the time series change from point to point. There are some reasons that direct calculation of the auto-correlation function doesn't often give the accurate exponent for the power law [11]. Determined fluctuation analysis is a common method for determining the scaling exponent of the auto-correlation function [11,12]. Using the DFA method we find an exponent, the so-called Hurst exponent, which exhibits the scaling behaviour of the auto-correlation function. Here we intend to find a reasonable relation between the Hurst exponent of a time series and the topological measures of its corresponding network. Hence in the following we describe a simple method for construction of complex networks from time series.

The difference between two adjacent values in a time series is called the return value, $r(t') = x(t') - x(t' - \Delta)$. The return series inherits the scaling properties of the original time series. By a simple rule a return series transforms to a sequence of characters [1], in this way any positive value in a return series is replaced by the character u and in the other way character d is substituted. Fig. 1 illustrates such a procedure. The resultant sequence can be partitioned into words of length L. There are 2^L distinct words that play the role of nodes of our network and a link between any two words is in place if they are adjacent to each other in the sequence. The network that is constructed by the described method may have weighted and directed links. Our method is somewhat similar to the procedure of Ref. [4] but simpler and faster. Besides we have complete control on the network size; the number of nodes and links, by determining the word's length L. It is crucial for rapid calculation of the topological measures of the network.

In Section 3 we apply this method to some model time series and compute the topological measures of the corresponding networks.