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Disorder induced phase transition in kinetic models of opinion dynamics

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ABSTRACT

We propose a model of continuous opinion dynamics, where mutual interactions can be both positive and negative. Different types of distributions for the interactions, all characterized by a single parameter p denoting the fraction of negative interactions, are considered. Results from exact calculation of a discrete version and numerical simulations of the continuous version of the model indicate the existence of a universal continuous phase transition at $p = p_c$ below which a consensus is reached. Although the order–disorder transition is analogous to a ferromagnetic–paramagnetic phase transition with comparable critical exponents, the model is characterized by some distinctive features relevant to a social system.

1. Introduction

Quantitative understanding of individual and social dynamics has been explored on a large scale [1–8] in recent times. Social systems offer some of the richest complex dynamical systems, which can be studied using the standard tools of statistical physics. With the availability of data sets and records on the increase, microscopic models mimicking these systems help in understanding their underlying dynamics. On the other hand, some of these models exhibit novel critical behavior, enriching the theoretical aspect of these studies.

Mathematical formulations of such social behavior have helped us to understand how global consensus (i.e., agreement of opinions) emerges out of individual opinions [9–25]. Opinions are usually modeled as variables, discrete or continuous, and are subject to spontaneous changes as well as changes due to binary interactions, global feedback and even external factors. Apart from the dynamics, the interest in these studies also lies in the distinct steady state properties: a phase characterized by individuals with widely different opinions and another phase with a major fraction of individuals with similar opinions. Often the phase transitions are driven by appropriate parameters of the model.

In this paper we study a model of opinion dynamics by considering two-agent interactions. Continuous opinion dynamics has been studied for a long time [26–28], with the models designed in such a way that eventually the opinions cluster around one (consensus), two (polarization) or many (fragmentation) values. The average opinion or macroscopic behavior has been emphasized only in some recent works [23,24], where a phase transition from ordered to disordered phase has also been reported. However, in contrast to these models, we obtain here an ordered phase where even in the presence of a dominant opinion (symmetry broken phase), opposing opinions survive and a disordered phase where all opinion values coexist without any preference to any value (symmetric phase). Thus we present this in the general context of an order–disorder transition similar to that of the Ising and related models. We also compare our results with earlier works where a mean-field phase transition was observed in presence of contrarians in the society [21].

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The paper is organized as follows: in Section 2 we introduce the model. Then in Section 3 the main results are presented along with the calculations and numerical simulations. In Section 4 we extend the model to include bond dilution and present the phase diagram. Finally we discuss our results in Section 5.

2. The model

We propose a new model for emergence of consensus. Let $o_i(t)$ be the opinion of an individual i at time t. In a system of N individuals (referred to as the 'society' hereafter), opinions change out of pair-wise interactions:

$$o_i(t+1) = o_i(t) + \mu_{ij} o_j(t). \tag{1}$$

One considers a similar equation for $o_j(t+1)$. The choice of pairs $\{i, j\}$ is unrestricted, and hence our model is defined on a fully connected graph, or in other words, of infinite range. Note that this is simply a pair-wise interaction and we imply no sum over the index j. Here μ_{ij} are real, and it is like an interaction parameter representing the influence of the individual with whom interaction is taking place. The opinions are bounded, i.e., $-1 \leq o_i(t) \leq 1$. This bound, along with Eq. (1) defines the dynamics of the model. If, by following Eq. (1) the opinion value of an agent becomes higher (lower) than $+1$ (-1), then it is made equal to $+1$ (-1) to preserve this bound. The ordering in the system is measured by the quantity $O = \left|\sum_i o_i\right| / N$, the average opinion, which is the order parameter for the system.

The present model is similar in form to a class of simple models proposed recently [23–25,29], apparently inspired by the kinetic models of wealth exchange [30,31]. A spontaneous symmetry breaking was observed in such models: in the symmetry broken phase, the average opinion is nonzero while in the symmetric phase, the opinions of all individuals are identically zero indicating a 'neutral state'. The parameters representing conviction (self interaction) and influence (mutual interaction) in these models were considered either uniform (a scalar) or in the generalized case different for each individual, i.e, given by the components of a vector. In addition to this there is an added feature of the randomness in the influence term which in effect controls the sharpness of the phase transitions in these models.

In our proposed model, the conviction parameter or self interaction parameter is set equal to unity so that in absence of interactions, opinions remain frozen. In such a situation, it has been observed previously that any interaction, however small, leads to a highly unrealistic state of all individuals having extreme identical opinions (either $o_i = 1\ \forall i$ or $o_i = -1\ \forall i$) [24] when the interactions take up *positive values only*. This suggests that one should generalize the interactions to include both positive and negative values. This is realistic also in the sense that it reflects the fact that in a social interaction of two individuals, there may be either agreement or disagreement of opinions. We therefore consider not only a distribution of the values of μ_{ij} (to maintain the stochastic nature of the interactions) but also allow μ_{ij} to have negative values. We define a parameter p as the fraction of values of μ_{ij} which are negative, which, we will show later, leads to characteristic ordered and disordered states as in reality.

The fact that we allow random positive and negative values for the interactions may suggest that the model is analogous to a dynamic spin glass model [32,33], as in the latter, one can consider a dynamic equation for the spins which formally resembles Eq. (1). However, the two dynamic models are not equivalent with the following differences: (i) the interactions in the opinion dynamics models are never considered simultaneously and thus the question of competition leading to the possibility of frustration does not arise, and (ii) there is also no energy function to minimize, (iii) the symmetry $p \to 1-p$ does not exist in our model, which is naturally present for spin-glass. We will get back to the comparison of the two models in the context of phase transition later in this paper.

The effect of negative interactions was considered previously in a different opinion dynamics model under the name Galam contrarian [21]. The discrete, binary opinion model followed a deterministic evolution rule for a group of three or more individuals. It was shown that depending on the concentration of the contrarians, the system will either reach an ordered state, where there one of the opinions will have majority, or a disordered state, where no clear majority is observed. The critical behavior of the model is similar to the one we present here at least in the fully connected graph. However, our model considers continuous opinion values. Also, the Galam contrarians always take the opinion opposite to that of the majority. However, in our case we also consider the present state of opinion of the agents and accordingly even the discrete version of our model has three states. A two-state discrete version of this model will not show any ordered state.

3. Results

Unless otherwise mentioned, we keep μ_{ij} values within the interval $[-1, 1]$ for simplicity. In principle, several forms can be considered for μ_{ij} (annealed, quenched, symmetric, non-symmetric etc.). Further, there can be several distribution properties for μ_{ij} in the interval $[-1, 1]$ (discrete, piecewise uniform and continuous distributions). Unless otherwise stated, in our study, we would discuss the case when μ_{ij} are *annealed*, i.e., they change with time. In other words, at each pairwise interaction, the value of μ_{ij} is randomly chosen respecting the fact that it is negative with probability p. For this case, the issue of symmetry does not arise. We consider distributions for both continuous and discrete μ_{ij}.

In all the above cases, we find a symmetry breaking transition. Below a particular value p_c of the parameter p, the system orders (i.e., the order parameter O has a finite non-zero value), while the disordered phase (where $O = 0$) exists for higher values of p. Since this phase transition is very much like the thermally driven ferromagnetic–paramagnetic transition in magnetic systems, we have considered the scaling of the analogous static quantities, which are: