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Compressible turbulence: Multi-fractal scaling in the transition to the dissipative regime

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ABSTRACT

Multi-fractal scaling in the transition to the dissipative regime for fully-developed compressible turbulence is considered. The multi-fractal power law scaling behavior breaks down for very small length scales thanks to viscous effects. However, the effect of compressibility is found to extend the single-scaling multi-fractal regime further into the dissipative range. In the ultimate compressibility limit, thanks to the shock waves which are the appropriate dissipative structures, the single-scaling regime is found to extend indeed all the way into the full viscous regime. This result appears to be consistent with the physical fact that vortices become more resilient and stretch stronger in a compressible fluid hence postponing viscous intervention. The consequent generation of enhanced velocity gradients in a compressible fluid appears to provide an underlying physical basis for the previous results indicating that fully-developed compressible turbulence is effectively more dissipative than its incompressible counterpart.

1. Introduction

Compressibility effects on fully-developed turbulence (FDT) are of importance in modern technological flow processes like makes of supersonic projectiles, hypersonic re-entry vehicles and high-speed internal flows in gas-turbine engines. Astrophysical processes like star formation in self-gravitating dense interstellar gas clouds via Jeans' instability [1,2] are other cases in point. So, considerable effort has been directed on this problem to date [3–17].

The multi-fractal model was used by Shivamoggi [9–11,14,15] to describe the numerically observed spatial intermittency in fully-developed compressible turbulence [4,5,18]. A multi-fractal object exhibits a global scaling structure that is described by a continuous spectrum of scaling exponents α and in a certain range $I \equiv [\alpha_{\min}, \alpha_{\max}]$. Each $\alpha \in I$ has the support set $S(\alpha) \subset \mathbb{R}^3$ of fractal dimension $f(\alpha)$ such that the velocity increment over a small distance ℓ has the scaling behavior [19],

$$\delta v(\ell) \sim \ell^{\alpha}, \quad \ell \text{ small.} \tag{1}$$

The sets $S(\alpha)$ are nested so that $S(\alpha') \subset S(\alpha)$ for $\alpha' < \alpha$. The fractal dimension $f(\alpha)$ is obtained via a Legendre transformation of the scaling exponent ξ_p of the *pth*-order structure function,

$$S_p(\ell) \equiv \langle |\delta v(\ell)|^p \rangle = \int_{\alpha_{\min}}^{\alpha_{\max}} \mathrm{d}\mu(\alpha) \ell^{\alpha p + 3 - f(\alpha)} \sim \ell^{\xi_p} \tag{2}$$

according to

$$\xi_p = \inf_{\alpha} [\alpha p + 3 - f(\alpha)], \quad \text{say for } \alpha = \alpha_* \tag{3}$$

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where,

$$\frac{\mathrm{d}f(\alpha^*)}{\mathrm{d}\alpha} = p. \tag{4}$$

The factor $(3 - f(\alpha))$ is the fraction of the coarse-graining measure of the volume of the set $S(\alpha)$ over the total volume.

When viscous dissipative effects arise, the multi-fractal power law described by Eq. (2) breaks down for very small ℓ and the cut-offs are determined by external parameters like the Reynolds number. In such cases, as in incompressible turbulence [20,21], the multi-fractal in question turns out to exhibit a pseudo-algebraic behavior with certain universal features, albeit on using a suitable rescaling via a multi-scaling method [20]. A similar situation arises also for a passive scalar diffusing in a random velocity field [22].

2. Multi-fractal scaling in the inertial range: behavior in the ultimate compressibility limit

In order to relate the fractal dimension $f(\alpha)$ to the generalized fractal dimension D_q of the kinetic energy dissipation field $\hat{\varepsilon}$, one considers a coarse-grained probability measure given by the total kinetic energy dissipation occurring in a box of size l and then covers the support of the measure $\hat{\varepsilon}$ with boxes of size ℓ and sums the moments $[E(\ell)]^q$ over all boxes. Noting the asymptotic scaling behavior of these moments [23]

$$\sum_{i=1}^{N(\ell)} [E(\ell)]^q \sim \ell^{(q-1)D_q} \tag{5a}$$

or

$$\int \mathrm{d}\mu(\alpha)\ell^{\left[\left(\frac{3\gamma-1}{\gamma-1}\right)\alpha+2\right]q-f(\alpha)} \sim \ell^{(q-1)D_q}. \tag{5b}$$

The polytrope exponent γ may be interpreted as a compressibility parameter[1] $(1 < \gamma < \infty)$—the incompressible fluid corresponds to the limit $\gamma \Rightarrow \infty$ while the ultimate compressibility case given by the limit $\gamma \Rightarrow 1$ corresponds to Burgers turbulence.

The dominant terms in the integral in (5b), in the limit of small ℓ, may again be extracted using the method of steepest descent:

$$\left[\left(\frac{3\gamma-1}{\gamma-1}\right)\alpha^* + 2\right]q - f(\alpha^*) = (q-1)D_q \tag{6}$$

with,

$$\frac{\mathrm{d}f(\alpha^*)}{\mathrm{d}\alpha} = \left(\frac{3\gamma-1}{\gamma-1}\right)q. \tag{7}$$

Eliminating $f(\alpha)$ and putting $q = \left(\frac{\gamma-1}{3\gamma-1}\right)p$, we obtain from (3) and (6) [8,9],

$$\xi_p = \left(\frac{\gamma-1}{3\gamma-1}\right)p - \frac{1}{3}\left[\left(\frac{3\gamma-3}{3\gamma-1}\right)p - 3\right]\left[3 - D_{\left(\frac{\gamma-1}{3\gamma-1}\right)p}\right]. \tag{8}$$

On noting the scaling behavior the density ρ [8] (which follows on applying scale-invariance arguments directly to the Navier–Stokes equations for a compressible fluid in conjunction with the scale-invariance condition on the kinetic energy dissipation field),

$$\rho \sim (\delta v)^{\frac{2}{\gamma-1}} \tag{9}$$

the scaling behavior of the kinetic energy is given by

$$\langle \rho(\delta v)^2 \rangle \sim \langle \delta v \rangle^{\frac{2\gamma}{\gamma-1}}. \tag{10}$$

(10), in conjunction with (2), leads to

$$\langle \rho(\delta v)^2 \rangle \sim \ell^{\xi\left(\frac{2\gamma}{\gamma-1}\right)} \tag{11}$$

[1] γ is like, but not quite, the ratio of specific heats of the fluid.