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Quantum game interpretation for a special case of Parrondo's paradox

Yong-fei Zhu, Neng-gang Xie *, Ye Ye, Fa-rui Peng

Department of Mechanical Engineering, Anhui University of Technology, Maanshan City, Anhui Province, 243002, China

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ABSTRACT

By using the discrete Markov chain method, Parrondo's paradox is studied by means of theoretical analysis and computer simulation, built on the case of game AB played in alternation with modulus $M = 4$. We find that such a case does not have a definite stationary probability distribution and that payoffs of the game depend on the parity of the initial capital. Besides, this paper reveals the phenomenon that "processing in order produces non-deterministic results, while a random process produces deterministic results". The quantum game method is used in a further study. The results show that the explanation of the game corresponding to a stationary probability distribution is that the probability of the initial capital has reached parity.

1. Introduction

Parrondo's paradox is a paradox in game theory and is named after its creator, J.M.R. Parrondo, a Spanish physicist. Parrondo's games present a case where given two games, each one with a higher probability of losing than of winning, it is possible to produce a winning outcome when they are played alternately or in a periodic order. The seminal papers concerning Parrondo's paradox were published by Harmer and Abbott in 1999 [1,2]. Already, Parrondo's paradox has been confirmed by means of computer simulation, the Brownian ratchet and discrete time Markov chain theory and has been developed into many different versions [3–8]. Flitney et al. also carried out an analysis of the quantum game based on Parrondo's paradox [9,10]. Although Parrondo's paradox is a counterintuitive phenomenon, where losing games combined can produce a winning result, similar phenomena can be found in many research areas [11–16]. Therefore, Parrondo's paradox is used in biology, physics and economics and so on.

Dr. Parrondo gave the initial version of the paradox game, which was composed of two associated, tossing biased coin games A and B, as shown in Fig. 1. Winning a game earns 1 unit and losing surrenders 1 unit.

(1) Game A is a game of tossing biased coin 1 with the probability of winning p_1.
(2) Game B is a little more complex. If the present capital is a multiple of some integer M, a biased coin 2 is tossed with probability of winning p_2; if not, another biased coin 3 is tossed, with probability of winning p_3.

Effectively choosing the values of probabilities p_1, p_2, p_3 and modulus M, playing games A and B individually would result in negative results. However, when two losing games are played in alternating or random sequences, this will lead to a winning result. Harmer et al. [17] carried out rigorous mathematical analysis of such a paradox by using recursive conditions for a discrete Markov chain and the theory of Shannon information entropy, and obtained the following conclusions:

* Corresponding author.
E-mail address: xienenggang@yahoo.com.cn (N.-g. Xie).

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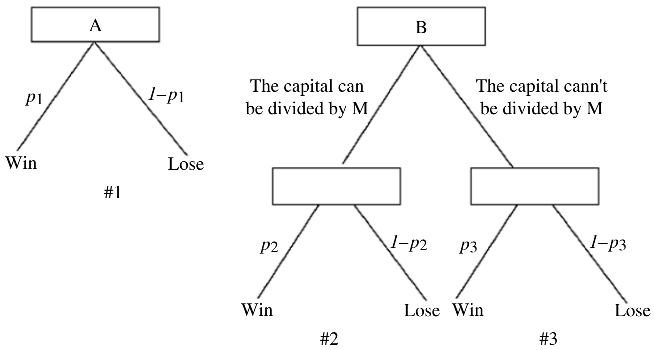

Fig. 1. Descriptions of games A and B.

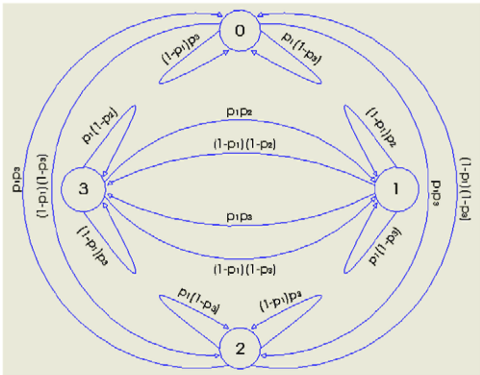

Fig. 2. Discrete time Markov chain of game AB defined by residual states.

Table 1
Possible changes in one step of AB.

State changes	010	030	012	032	121	101	123	103
Probabilities	$p_1(1-p_3)$	$(1-p_1)p_3$	p_1p_3	$(1-p_1)(1-p_3)$	$p_1(1-p_3)$	$(1-p_1)p_2$	p_1p_3	$(1-p_1)(1-p_3)$
State changes	230	210	232	212	301	321	303	323
Probabilities	p_1p_3	$(1-p_1)(1-p_3)$	$p_1(1-p_3)$	$(1-p_1)p_3$	p_1p_2	$(1-p_1)(1-p_3)$	$p_1(1-p_2)$	$(1-p_1)p_3$

(1) when $p_1 < 0.5$, game A will produce a losing result;
(2) for an arbitrary case with $M \geq 3$, if

$$p_2 < \frac{(1-p_3)^{M-1}}{p_3^{M-1} + (1-p_3)^{M-1}}, \tag{1}$$

then game B will produce a losing result;
(3) for an arbitrary case of a random combination of playing games A and B, when $M \geq 3$, if $q_2 > \frac{(1-q_3)^{M-1}}{q_3^{M-1}+(1-q_3)^{M-1}}$, the result of randomly playing games A and B is winning, where $q_2 = \gamma p_1 + (1-\gamma)p_2$, $q_3 = \gamma p_1 + (1-\gamma)p_3$ and parameter γ is the probability of playing game A.

2. The theoretical analysis of game AB

Let game A and game B be played one time, together, as a step. If the capital at time t is $X(t)$, $Y(t) = X(t) \bmod 4$, then the state set of the residue $Y(t)$ is $E = \{0, 1, 2, 3\}$. The discrete Markov chain defined by the states of the residue $Y(t)$ can be seen in Fig. 2, where clockwise is a winning direction. From steps t to $t+1$, there are only 16 kinds of state changing processes of residue $Y(t)$, as shown in Table 1.

On the basis of Table 1 and Fig. 2, the transition probability matrix is as follows:

$$\mathbf{P} = \begin{bmatrix} p_1 - 2p_1p_3 + p_3 & 0 & 1 - p_1 - p_3 + 2p_1p_3 & 0 \\ 0 & p_1 + p_2 - p_1p_3 - p_1p_2 & 0 & 1 - p_1 - p_2 + p_1p_2 + p_1p_3 \\ 1 - p_1 - p_3 + 2p_1p_3 & 0 & p_1 - 2p_1p_3 + p_3 & 0 \\ 0 & 1 - p_1 - p_3 + p_1p_3 + p_1p_2 & 0 & p_1 - p_1p_2 + p_3 - p_1p_3 \end{bmatrix}. \tag{2}$$