Physica A 390 (2011) 587–594

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Nonadditive Tsallis entropy applied to the Earth's climate

J.L. Gonzalez [a,*], E.L. de Faria [b], Marcelo P. Albuquerque [b], Marcio P. Albuquerque [b]

[a] *Departamento de Física, Universidade Federal do Espírito Santo, Vitória, ES, Brazil*
[b] *Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, RJ, Brazil*

ARTICLE INFO

Article history:
Received 14 September 2010
Available online 3 November 2010

Keywords:
Nonadditive Tsallis entropy
Earth's climate
Complex systems
Time series

ABSTRACT

The concepts of nonextensive statistics, which has been applied in the study of complex systems, are used to analyze past records of the Earth's climate. The fluctuations within the record of deuterium content (hence temperature) in the last glacial period appear to follow a q-Gaussian distribution. Analyses of the time-dependent nonadditive entropy indicate transitions between different complexity levels in the data prior to the abrupt change in the system dynamics at the end of the last glaciation. Different fluctuation regimens are evidenced through wavelets analysis. It is also suggested that time-dependent entropy analysis could be useful for indicating the approach to a critical transition of the Earth's climate for which theoretical models are in many cases not available.

1. Introduction

Nonlinear systems are present in different areas of knowledge, like ecology, medicine, economics and engineering. Within these disciplines, and others, chaotic dynamics and complex behavior usually appear, leading to the emergence of new features and states with high levels of organization. In these systems, the existence of thresholds and that of crossovers are common features. Examples have been detected in epileptic seizures in medicine, financial crashes, catastrophic shifts in ecosystems, to name but a few [1–3]. Analyzing the different levels of complexity is essential to understanding the system's dynamics and to predicting its behavior using evidence such as fingerprints, or early-warning signals, of critical changes in the time series [4–9].

Critical transitions or abrupt changes in the past temperature records of the Earth are among the most striking examples of complexity emergence [9–11]. On the other hand, for the Earth's climate, like for many other natural systems, the models determining the dynamics of the system are unknown, and information is usually obtained from the analysis of highly nonlinear time series (TS) [4,11]. Entropy analysis is one of the complexity measurements or statistical tools that can provide information about the dynamics of a specific system. This analysis has been used to separate healthy from pathological physiological time series [12], in the quantification of spatio-temporal dynamics of the human brain [13], and in similar matters [14].

On the other hand, natural systems, like the Earth's climate, usually have non-equilibrium stationary states whose dynamics can, in some cases, be conveniently addressed through nonextensive statistical mechanics [15]. Indeed, Tsallis statistics has been satisfactorily applied to describe changes in complexity of various systems, such as electroencephalograms [16], detection of different levels of organization (complexities) between periods of intense and normal magnetic activity in the Earth's magnetosphere [17], description of the number of citations by the Institute of Scientific Information [18], and many other recent works [19–24].

* Corresponding address: Departamento de Física, Universidade Federal do Espírito Santo, Av. Fernando Ferrari 514, Goiabeiras, Vitória, Espírito Santo, CEP: 29075 910, Brazil. Tel.: +55 27 40097727.
E-mail address: jlga.1966@gmail.com (J.L. Gonzalez).

0378-4371/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2010.10.045

The present work uses the nonadditive entropy, on which nonextensive statistical mechanics is based, for analyzing the climate fluctuations in past deuterium records corresponding to the last glacial period, which characterize the climate variability history. The principal goal here is to show that the nonextensive Tsallis statistics allows for a convenient analysis of the climate. Consistently we will identify different complexity levels across the time series of past deuterium content fluctuations, pointing out the suitability of the Tsallis entropy for describing climate transformations.

2. Tsallis entropy

The nonadditive Tsallis entropy is determined by

$$S_q \equiv k \sum_{1}^{W} p_i \ln_q \left(\frac{1}{p_i}\right) \equiv \frac{k}{q-1}\left(1 - \sum_{1}^{W} p_i^q\right). \tag{1}$$

In Eq. (1), $\ln_q(x) = \frac{x^{1-q}-1}{1-q}$ is the q-logarithm function. S_q according to Eq. (1) is a generalization of the Boltzmann–Gibbs (BG) entropy $\left(S_{BG} = -k_B \sum_{i=1}^{W} p_i \log p_i\right)$ [25], and it is essentially extensive when applied to nonlinear scale-invariant systems with long range correlations [25,26]. In the last expression, p_i is the probability of a microscopic configuration, W being the number of microstates (configurations) and the parameter $q \in \Re$ characterizes the nonextensivity of the system. For $q = 1$ it reproduces the BG result, while it is sub-extensive for $q > 1$ and super-extensive for $q < 1$.

Extremization of the continuous version of S_q with appropriate constraints leads to the q-Gaussian distribution which is based on the q-exponential function: $e_q(x) \equiv (1 + (1-q)x)^{\frac{1}{1-q}}$ whenever the argument is positive, and zero otherwise. Note that for $q = 1$ the e_q function reduces to the common $\exp(x)$ function. The q-Gaussian distributions can be written using the q-exponential as follows:

$$G_q(x) = A_q \left(1 + \frac{(q-1)x^2}{(3-q)\sigma_q}\right)^{\frac{1}{1-q}} \tag{2}$$

where σ_q is the q-generalized variance (a convenient characterization of the width of the distribution), and A_q is a normalization constant with values

$$A_q = \begin{cases} \dfrac{\Gamma\left(\frac{5-3q}{(2-2q)}\right)}{\Gamma\left(\frac{2-q}{(1-q)}\right)}\sqrt{\dfrac{(1-q)}{\pi(3-q)\sigma_q}} & q < 1 \\ \sqrt{\dfrac{1}{\pi(3-q)\sigma_q}} & q = 1 \\ \dfrac{\Gamma\left(\frac{1}{(q-1)}\right)}{\Gamma\left(\frac{3-q}{(2q-2)}\right)}\sqrt{\dfrac{(q-1)}{\pi(3-q)\sigma_q}} & q > 1. \end{cases} \tag{3}$$

3. Earth's climate and complexity

The Earth's climate is a natural complex system with many variables and feedback mechanisms leading to emerging properties that are not easy to compare with the available theoretical models. Time series of deuterium content (which characterizes the temperature) variations show clear evidence of abrupt changes which have been taken as critical thresholds or tipping points [11]. Examples are the greenhouse–icehouse transition [27], the abrupt change at the end of the Younger Dryas cold period [28], the end of glaciations [29], among others [4].

In ecosystems, through the study of fluctuations, it is possible to identify regularities in the time series data that may throw some light on the mechanisms involved [30,31]. Fluctuations analysis is also crucial to the *critical slowing down* phenomenon which consists in the fact that the necessary time for returning to equilibrium increases, after a system is perturbed, as the critical point is approached [4]. Following this idea, Kleinen modeled the hemispheric thermohaline circulation system through a simple stochastic model which exhibits changes in the power spectral density as the system moves toward the critical region [32]. Held and Kleinen considered the climate as a system at equilibrium, stochastically perturbed by noise, whose dynamics is determined by the most unstable mode (the critical mode with the smallest decay rate). This mode disappears at the critical threshold causing the fluctuations to follow a 1D autocorrelation (AR-1) process. This idea was satisfactorily tested for the North Atlantic thermohaline circulation system [5]. Also, an increase in the variance of the fluctuations can be taken as a signal of a critical point approaching [33].

Other studies point to non-Gaussian character of the Earth's temperature fluctuations, suggesting that the Earth's climate should be considered as a non-equilibrium system [30]. In this sense, evidences of complexity exist linking the dynamics of the Earth's climate and that of the Sun, where complex events like solar flares, sunspots and others [31] occur. It seems