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Non-equilibrium stochastic model for stock exchange market

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HIGHLIGHTS

- The effect of the industrial relationship (IR) on financial market is studied.
- We model the financial market based on the behavior of technical traders.
- We measure the return distribution and autocorrelation function of volatility.
- The heterogeneous IR topology is a possible origin of the universal features.

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ABSTRACT

We study the effect of the topology of industrial relationship (IR) between the companies in a stock exchange market on the universal features in the market. For this we propose a stochastic model for stock exchange markets based on the behavior of technical traders. From the numerical simulations we measure the return distribution, $P(R)$, and the autocorrelation function of the volatility, $C(T)$, and find that the observed universal features in real financial markets are originated from the heterogeneity of IR network topology. Moreover, the heterogeneous IR topology can also explain Zipf–Pareto's law for the distribution of market value of equity in the real stock exchange markets.

1. Introduction

Financial markets have been increasingly investigated in statistical physics as a part of complex systems [1–3]. Empirical studies based on various financial market data have revealed very interesting universal properties which govern the dynamical properties of financial markets. For example, the probability distribution of the price or the market index return, and the autocorrelation function of the volatility have revealed interesting universal features in financial markets [3–5]. Such universal features are usually called as the stylized facts. Thus, uncovering the origin of universal behaviors observed in various financial markets is important to understand the interesting behaviors in financial markets. For this, many stochastic and agent-based models have been proposed [2,6–13].

Since there are no apparently tunable external parameters in financial markets, some studies have focused on the self-organized criticality of financial systems [14,15]. The self-organizing features are frequently observed in various socio- and econo-systems [16,17] as well as biological [18] and technological systems [19] in which cascade or avalanche plays a crucial role. The self-organized criticality is usually characterized by the punctuated equilibrium [20], in which the time series show intermittent occurrences of large bursts separated by relatively long periods of quiescence. In financial time series, the price return is one of the well-known examples for such intermittent occurrences of large bursts. This usually causes the volatility clustering [4] which plays a very crucial role in modeling financial markets.

In this paper, we propose a simple stochastic model which incorporates the behavior of agents in financial markets. In the real market, there are two types of investors. The first are fundamentalists who attempt to determine the fundamental values of stocks. The second are technical traders who make their trading decisions based on the price pattern. Although

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the fundamentalists hold a majority of the stocks, the contribution of the technical traders to the market dynamics is much larger than that of the fundamentalists because of the frequent trading activity of the technical traders [11]. Since the trading decisions of technical traders are based on the price pattern, they can make somewhat coherent decisions. Such coherent behavior of agents is usually called a herd behavior [12,21,22]. Recently we showed that the herd behavior of the agents in a financial market causes the power-law scaling of tails in return distribution and is also related to the economic crisis [22]. In this paper we study the effects of the coarse grained behavior of technical traders and the topology of the industrial relationship (IR) between companies in financial markets on the market dynamics. As we shall show, one possible origin of the observed stylized facts in stock exchange markets is the topology of underlying IR network between the listed companies in stock exchange markets. To verify this we use four different underlying structures, 1*D* and 2*D* regular lattices, random networks, and scale-free networks.

2. Price update rule

The herd behavior of technical traders is frequently observed from the result of coherent decisions in a group of agents to buy (or sell) stocks of a specific company [22]. However, some of the agents in the group would not want to buy (or sell) the stocks if the price of the stock becomes higher (or lower) than their expectations. In such a case, those agents would seek other companies to buy (or sell) stocks. The possible alternatives are the companies which have industrial relationship to the selected company. For example, the prices of stocks of oil companies usually increase or decrease together. Thus, if the stock price of Exxon Mobile becomes too high (low), then some agents would seek other oil companies to buy (sell) their stocks. To incorporate such features into the model we define the following stochastic procedures for stock exchange markets.

Let N be the number of companies listed in a stock exchange market. The price of a stock of ith company at time t is denoted by $p_i(t)$. $p_i(t)$ evolves initially from the same scaled prices $p_i(0) = p_0$ for all i. Here p_0 is a constant and set to be zero in the following simulations. The price is updated by the following steps. (i) Select a company i at random. (ii-1) Increase the price of company i by random increment $\Delta(t)(\in (0, 1])$ with a probability P $(p_i(t) \to p_i(t) + \Delta(t))$. This mimics the increase of stock price by a buying decision of a group of agents. (ii-2) Then check the price differences between i and all of its nearest neighbors, j's. (ii-3) For a preassigned positive constant C, if $p_i(t) - p_j(t) \geq C$, then adjust the price of stocks of neighboring companies j upwards $(p_j(t) \to p_j(t) + \Delta(t))$. This process reflects the fact that the price of i is too high to buy; thus the agents would seek other companies which have industrial relationship to the company i with underestimated price. (ii-4) Repeat the processes (ii-2)–(ii-3) for all nearest neighbors of j's until prices of all companies and their nearest neighbors satisfy $p_i(t) - p_j(t) < C$. (iii-1) With the probability $1 - P p_i(t)$ decreases by $\Delta(t)$ $(p_i(t) \to p_i(t) - \Delta(t))$ if $p_i(t) - \Delta(t) \geq 0$. The imposed condition $p_i(t) - \Delta(t) \geq 0$ implies that the price should be positive and provides the temporal minimum of $p_i(t)$. Thus, the agent would not sell their stocks when $p_i(t)$ is equal to the temporal minimum, because they would expect that the stock price of i is underestimated and will increase in near future. (iii-2) Calculate $p_j(t) - p_i(t)$ for all its nearest neighbors j's. (iii-3) If $p_j(t) - \Delta(t) > 0$ and $p_j(t) - p_i(t) \geq C$, then adjust $p_j(t)$ downwards $(p_j(t) \to p_j(t) - \Delta(t))$. (iii-4) Repeat (iii-2)–(iii-3) until all prices of companies satisfy $p_j(t) - p_i(t) \geq C$. This corresponds to the decrease of stock price by the coherent selling decisions. The processes (ii-4) and (iii-4) cause the successive updates of $p_i(t)$'s, which are called as an avalanche [20].

This update rule is reminiscent of Senppen's model in the interface roughening phenomena [23] or innovation propagation model in sociophysics [24]. Since the value of C only affects the scale of price, we set $C = 1$ in the following simulations for simplicity. We also set $P = 1/2$, because if $P < 1/2$, then the average price decreases and when $P > 1/2$ the average price linearly increases as t. The unit time is defined as the usual Monte Carlo time step. For the direct comparison with real market indices, we use $N = 1024$.

3. Underlying structures

To study the effect of the structure of IR on market properties, we consider two different IR network topologies, random network (RN) and scale-free network (SFN), as well as 1-dimensional (1*D*) and 2-dimensional (2*D*) regular lattices. Here the number of industrial relationships of a company i corresponds to the degree, k_i, of the company. The degree distribution of RN is generally known to be the Poisson distribution, which means that the degree distribution for RN is homogeneous. To generate RNs we use the Erdös–Rényi network model [25]. In contrast to RN, the degree distribution of SFNs is highly heterogeneous which is characterized by a power-law, $P(k) \sim k^{-\gamma}$. In the following simulations we use the Goh et al.'s model to implement SFN for underlying topology [26]. In this SFN model, a weight $w_i = i^{-\alpha}$ is assigned to each node i $(i = 1, 2, \ldots, N)$, where $0 \leq \alpha < 1$. By adding a link between unconnected nodes i and j with probability $w_i w_j / (\sum_{n=1}^{N} w_n)^2$, one can obtain a network whose degree distribution satisfies a power-law $P(k) \sim k^{-\gamma}$. In this SFN model, γ is related to α as $\gamma = (1 + \alpha)/\alpha$. Thus, by adjusting α we obtain a network with any γ (> 2).

4. Market index

The market index is a measure of aggregated market value and is usually defined by the weighted average of market value of equities of listed companies [27]. Thus, the companies which have high price and large number of outstanding