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A decentralized flow redistribution algorithm for avoiding cascaded failures in complex networks

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HIGHLIGHTS

- A decentralized algorithm for avoiding cascaded failures in complex networks is proposed.
- The algorithm is based on information about the closest neighbours of each node.
- The algorithm is implemented locally and does not need a centralized control system.
- A mathematically rigorous proof of convergence with probability 1 is provided.
- The maximum flow and the constant flow supply/demand problems are also considered.

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ABSTRACT

A decentralized random algorithm for flow distribution in complex networks is proposed. The aim is to maintain the maximum flow while satisfying the flow limits of the nodes and links in the network. The algorithm is also used for flow redistribution after a failure in (or attack on) a complex network to avoid a cascaded failure while maintaining the maximum flow in the network. The proposed algorithm is based only on the information about the closest neighbours of each node. A mathematically rigorous proof of convergence with probability 1 of the proposed algorithm is provided.

1. Introduction

The maximum flow problem has been a classical research topic in the field of complex networks. Since Ford and Fulkerson [1] introduced their augmenting paths algorithm in 1956, many algorithms have been developed to study and solve this problem [2–10]. Applications of the maximum flow problem include the determination of the maximum steady-state flow of petroleum products in pipelines, cars on roads, messages in telecommunication networks, and electricity in electrical grids [11]. It is also used in computer vision problems [12] and World Wide Web analysis [13].

Several research works developed decentralized algorithms to solve the maximum flow problem. These algorithms were mainly decentralized versions of originally centralized algorithms. For example, Segall [14] proposed three algorithms based on Refs. [1,2,15], Cheung [16] proposed an algorithm based on Ref. [1], and Awerbuch [17] proposed an algorithm based on Ref. [18]. These algorithms assume that the network contains only one source and one sink. As a result the decentralized versions are limited to single-source single-sink networks.

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In a typical centralized algorithm, multi-source multi-sink networks are transformed into a single-source single-sink network and then the maximum flow is computed. Obviously, this transformation is not extendable to decentralized algorithms. Although recent publications considered the maximum flow problem in multi-source multi-sink planer networks without this transformation [12,19], the information regarding the status of all the nodes and the links in the network are assumed to be readily available for a centralized system to apply the algorithm. On the contrary, the algorithm proposed in this paper is natively decentralized and capable of handling multi-source multi-sink networks.

In addition, the flow in the network should be distributed such that the maximum capacities of the network elements are not exceeded. Otherwise, the network would potentially be vulnerable to cascaded failures. Cascaded failures in complex networks have been the subject of many studies [20,21] including, but not limited to, cascaded failures in urban traffic networks [22–24] and power grids [25,26].

In electrical power distribution systems, for example, cascaded failures have been attracting the attention of researchers due to the catastrophic impact they have on the networks [27–29]. A cascaded failure is initiated when a node or a link in the complex network is lost due to a random failure or a targeted attack, and the flow passing through the lost node or link is redistributed to other nodes and links in the network. If not executed properly, the redistribution may cause other nodes and links to become overloaded and therefore disconnected from the network. This effect may propagate throughout the network and the entire network may be affected [30].

Cascaded failures were the reason for large blackouts throughout the world (e.g. Refs. [31–34]). As a consequence, researchers have been developing control algorithms for power flow redistribution with minimum load loss [35,30,36,37]. Typically, the maximum power flow that can be delivered to the loads is calculated by applying linear programming which has been used since the 1970s [35,38–40,36]. However, similar to the maximum flow algorithms, linear programming requires the status of all the nodes and the links in the network to be available for a centralized control system.

Switched networks [41,42] are another type of networks that are controlled by a single server that operates locally on one node at a time, but the location of the server is a control variable that is determined using information from the entire network represented by a hybrid dynamical system [43].

The new algorithm proposed in this paper to drive the flow distribution in a complex network aims to avoid cascaded failure in the network while maintaining the maximum flow. The novel decentralized randomized control algorithm is based on simple local control rules that are based only on information about the closest neighbours of each node in the network. Unlike the algorithms discussed above where a central controller is required, the proposed algorithm is implemented locally at each node. As a result, the computational costs are reduced and less information needs to be transferred on the network.

In addition, the algorithm is directly applicable to multi-source multi-sink networks. Another advantage of the proposed algorithm is that the flow values and capacities in the network can be real numbers and not limited to integer values.

The proposed algorithm is theoretically verified. In particular, this paper provides a mathematically rigorous proof of convergence of the algorithm with probability 1 for any initial flow distribution. The behaviour of the algorithm is described by an absorbing Markov chain and the algorithm is shown to converge to one of the absorbing states. Markov chains with absorbing states are already being used to model networks [44,45]. The randomized algorithm used in this paper is inspired by the distributed random algorithm used in Ref. [46] for blanket coverage self-deployment of a network of mobile wireless sensors.

The remainder of this paper is organized as follows: the flow redistribution problem addressed in this paper is formulated in the next section along with two special cases. The proposed control algorithm is detailed in Section 3 where the algorithm is proved to converge to a solution with probability 1 and a summary of the simulation procedure is provided. Simulation results are shown in Section 4, followed by the conclusion in Section 5.

2. Problem statement

Consider a complex network modelled by a graph $G = (\mathcal{N}, \mathcal{L})$ defined by a set $\mathcal{N}$ of n nodes connected by a set $\mathcal{L}$ of l links. For a given time k, each node $i \in \mathcal{N}$ is associated with a finite real value $N_i(k)$ that corresponds to the actual net flow in the node and represents the flow supply or demand of the node. The node i is a source (supply) node if

$$0 < N_i(k) \leq N_{i_{\max}} \tag{1}$$

where $N_{i_{\max}}$ is a predefined positive value representing the maximum capacity of the source node i to supply flow. The node i is an intermediate (transshipment) node if

$$N_i(k) = 0. \tag{2}$$

The node i is a sink (demand) node if

$$N_{i_{\min}} \leq N_i(k) < 0 \tag{3}$$

where $N_{i_{\min}}$ is a predefined negative value representing the maximum capacity of the sink node i to demand flow.

Each link $\langle i, j\rangle \in \mathcal{L}$ connecting the two nodes i and j is assigned a flow value $F_{i,j}(k)$ that should not exceed a maximum capacity

$$|F_{i,j}(k)| \leq F_{i,j_{\max}}; \tag{4}$$