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Measuring the self-similarity exponent in Lévy stable processes of financial time series

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HIGHLIGHTS

- We study two previously introduced algorithms to measure the self-similarity exponent of Lévy stable motions.
- We compare its performance with another known algorithms.
- We apply them to study memory in stock prices.

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ABSTRACT

Geometric method-based procedures, which will be called **GM algorithms** herein, were introduced in [M.A. Sánchez Granero, J.E. Trinidad Segovia, J. García Pérez, Some comments on Hurst exponent and the long memory processes on capital markets, Phys. A 387 (2008) 5543-5551], to efficiently calculate the self-similarity exponent of a time series. In that paper, the authors showed empirically that these algorithms, based on a geometrical approach, are more accurate than the classical algorithms, especially with short length time series. The authors checked that **GM algorithms** are *good* when working with (fractional) Brownian motions. Moreover, in [J.E. Trinidad Segovia, M. Fernández-Martínez, M.A. Sánchez-Granero, A note on geometric method-based procedures to calculate the Hurst exponent, Phys. A 391 (2012) 2209-2214], a mathematical background for the validity of such procedures to estimate the self-similarity index of any random process with stationary and self-affine increments was provided. In particular, they proved theoretically that **GM algorithms** are also valid to explore long-memory in (fractional) Lévy stable motions.

In this paper, we prove empirically by Monte Carlo simulation that **GM algorithms** are able to calculate accurately the self-similarity index in Lévy stable motions and find empirical evidence that they are more precise than the absolute value exponent (denoted by **AVE** onwards) and the multifractal detrended fluctuation analysis (**MF-DFA**) algorithms, especially with a short length time series. We also compare them with the generalized Hurst exponent (**GHE**) algorithm and conclude that both **GM2** and **GHE** algorithms are the most accurate to study financial series. In addition to that, we provide empirical evidence, based on the accuracy of **GM algorithms** to estimate the self-similarity index in Lévy motions, that the evolution of the stocks of some international market indices, such as **U.S. Small Cap** and **Nasdaq100**, cannot be modelized by means of a Brownian motion.

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1. Introduction

In finance, the most important theories such as market efficiency, classical portfolio or the capital asset pricing model, have been developed under the hypothesis of normality in the evolution of trends. The starting point of all these theories began during the 1940s with the application of Louis Bachelier's works and concluded with the first anthology about capital market behavior compiled by Cootner in Ref. [1]. This work collects the fundaments of the efficient market theory introduced in Refs. [2,3]. Since the publication of this anthology, most financial models assume that stock returns can be modelized by independent and identically distributed (denoted i.i.d. herein) random variables and that the distribution that follows is the normal one.

However, in Ref. [4], Fama showed that the empirical distribution of stock market returns is leptokurtic and has heavy tails in comparison with the normal distribution. Following Fama's steps, at the end of sixties, several authors pointed out further deficiencies of the Gaussian distribution to describe stock returns behavior, primarily due to the presence of kurtosis and skewness in financial time series. Since then several distributions have been proposed to model stock market returns.

Both the normal and the log-normal distributions have been widely used in financial literature as a good fit for financial series, mainly because the estimation of their parameters becomes relatively simple and the (log-)normal distribution is rather convenient for modeling. As McDonald noticed (see Ref. [5]), although these distributions provide appropriate descriptive models in most cases, many time series exhibit some characteristics intractable from the point of view of these two distributions.

Another distribution widely used in the literature is the Student's one. This distribution has fatter tails than the normal distribution but it is not helpful for skewness. The use of this distribution is mainly motivated because when the number of degrees of freedom is greater than 30, then the addition of Student's distributions converges to a normal one as a consequence of the Central Limit Theorem. Furthermore, Blattberg and Gonnedes obtained that the number of degrees of freedom for most securities exceeded 25 for sum sizes of 20, corresponding to monthly returns (see Ref. [6]). Since a Student's distribution with 25 degrees of freedom is *almost* a normal distribution, then the use of the Student's distribution does not seem to be very helpful.

Another extended practice has been the use of more flexible parametric distributions, namely, families of distributions which include many common distributions as particular cases. Examples are Lévy processes, which have long been used in finance. The first to propose an exponential non-normal Lévy process was Mandelbrot in 1963 (see Ref. [7]). He observed that the logarithm of relative price changes on financial and commodity markets exhibit a long-tailed distribution. He proposed to replace the Brownian motion based model by a model based on a symmetric α-stable Lévy motion with parameter $\alpha < 2$. This yields a pure-jump stock-price process. In particular, note that normal distributions are α-stable distributions with $\alpha = 2$. A few years later, an exponential Lévy process model with a non-stable distribution was proposed by Press in Ref. [8]. His log-price process is a superposition of a Brownian motion and an independent compound Poisson process with normally distributed jumps. Other examples were developed in Ref. [9], where a Lévy process with gamma variance distributed increments was proposed as a model for log-prices. Like α-stable Lévy motions, gamma variance Lévy processes are pure-jump processes. Gamma variance distributions are limiting cases of the family of generalized hyperbolic distributions which were introduced by Barndorff-Nielsen in Ref. [10] as a model for the grain-size distribution of wind-blown sand.

Two subclasses of the generalized hyperbolic distributions have been shown to provide an excellent fit to empirically observed log-return distributions: Eberlein and Keller introduced an exponential hyperbolic Lévy motion as a stock price model [11], and Barndorff-Nielsen proposed an exponential normal inverse Gaussian Lévy process (see Ref. [10]). The whole family of generalized hyperbolic Lévy processes was finally studied by Eberlein and Prause in Ref. [12].

It is also interesting to quote Kozubowski [13], where geometric stable laws are introduced as a new kind of alternative stable distributions, Kozubowski–Panorska [14], where the multivariate geometric stable distribution is used to model financial portfolios and finally, Kozubowski–Podgórski (see Ref. [15]), where an asymmetric Laplace, which is a subclass of geometric stable distributions, is proposed to model exchange rates.

To conclude, Koponen [16], Boyarchenko and Levendorskii [17], and Carr et al. [18], developed the Classical Tempered Stable distribution (CTS); Kim et al., the Modified Tempered Stable (MTS) (see Ref. [19]) and finally, Kim et al. presented an interesting application of the KR distribution, which is a subclass of the Tempered Stable distribution, using a GARCH model (see Ref. [20]).

Stable families for asset returns have been rejected for several reasons. Firstly, the efficient estimation of parameters is well-known in limiting cases. The second reason is given by a series of articles where the Lévy stability of returns is rejected based on the log–log linear regression for the cumulative function or the Hill estimator (see Refs. [18,19,21]). Regarding to the first disadvantage, today there exist reliable computer programs to calculate Lévy stable density distribution functions and their quantiles (see Refs. [22–24]). The second disadvantage has been dealt with by Weron [25]. The author shows that widely used tail index estimators can give exponents above the asymptotic limit for Lévy stable distributions with α close to 2. The author also shows that tail indices are significantly overestimated in samples of typical size but not in large ones. This problem may cause erroneous rejection of the Lévy stable hypothesis.

However, in Ref. [26] it was found that an asymmetric Lévy probability distribution function (PDF for short) allows us to model several multiple credit ratios applied in financial accounting to quantify a firm's financial health. In particular, asymmetric Lévy PDFs model both the Altman Z score and the Zmijewski score, and also the changes of individual financial ratios.