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Time-reversal asymmetry in financial systems

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HIGHLIGHTS

- We investigate large-fluctuation dynamics in financial markets.
- The dynamics is time-reversal symmetric in time scales of minutes.
- The dynamics is time-reversal asymmetric at daily time scales.
- Time-reversal asymmetry is induced by exogenous events.

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ABSTRACT

We investigate the large-fluctuation dynamics in financial markets, based on the minute-to-minute and daily data of the Chinese Indices and the German DAX. The dynamic relaxation both before and after the large fluctuations is characterized by a power law, and the exponents $p_{\pm}$ usually vary with the strength of the large fluctuations. The large-fluctuation dynamics is time-reversal symmetric at the time scale in minutes, while asymmetric at the daily time scale. Careful analysis reveals that the time-reversal asymmetry is mainly induced by external forces. It is also the external forces which drive the financial system to a non-stationary state. Different characteristics of the Chinese and German stock markets are uncovered.

1. Introduction

Financial markets are complex systems which share common features with those in traditional physics. In recent years, large amounts of high-frequency data have piled up in stock markets. This allows an analysis of the fine structure and interaction of the financial dynamics, and many empirical results have been documented [1–11]. Although the price return of a financial index is short-range correlated in time, the volatility exhibits a long-range temporal correlation [2,3]. The dynamic behavior of volatilities is an important topic in econophysics [2,3,12,13].

Assuming that a financial market is in a stationary state, one may analyze its static statistical properties. For a comprehensive understanding of the financial market, however, it is also important to investigate the non-stationary dynamic properties. A typical example is the so-called financial crash [14,6]. Lillo and Mantegna study three huge crashes of the stock market, and find that the rate of volatilities larger than a given threshold after such market crashes decreases by a power law with certain corrections in shorter times [15]. This dynamic behavior is analogous to the classical Omori law, which describes the aftershocks following a large earthquake [16]. Selcuk analyzes the daily data of the financial indices from 10 emerging stock markets and also observed the Omori law after the two largest crashes [17]. Mu and Zhou extend such an analysis to the stock market in Shanghai [18]. Recently, Weber et al. demonstrate that the Omori law holds also after "intermediate shocks", and the memory of volatilities is mainly related to such relaxation processes [19].

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Stimulated by these works, we systematically analyze the *large-fluctuation* dynamics in financial markets, based on the minute-to-minute and daily data of the Chinese Indices and German DAX. In our study, a large fluctuation is identified when its volatility is sufficiently large compared with the average one. At the time scale in minutes, those large volatilities may have nothing to do with real financial crashes or *rallies*, and represent only extremal fluctuations at the microscopic level. Even at the daily time scale, a large volatility maybe also not yet corresponding to a real financial crash or rally. But as the magnitude of the large volatility increases, it approaches a real financial crash or rally. In fact, the dynamic behavior of rallies has not been touched to our knowledge.

The purpose of this paper is multi-fold. We investigate the dynamic relaxation both *before* and *after* the large fluctuations. We focus on the time-reversal symmetry or asymmetry at different time scales. To achieve more reliable results, we introduce the remnant and anti-remnant volatilities to describe the large-fluctuation dynamics, different from those in Refs. [15,17,18]. More importantly, we examine the dynamic behavior of different categories of large fluctuations, and explore the origin of the time-reversal asymmetry at the daily time scale. We reveal how the financial system is driven to a non-stationary state by exogenous events, and compare the results of the mature German market and emerging Chinese market.

2. Large-fluctuation dynamics

In this paper, we have collected the daily data of the German DAX from 1959 to 2009 with 12 407 data points, and the minute-to-minute data from 1993 to 1997 with 360 000 data points. The daily data of the Shanghai Index are from 1990 to 2009 with 4 482 data points, and the minute-to-minute data are from 1998 to 2006 with 95 856 data points. The daily data of the Shenzhen Index are from 1991 to 2009 with 4 435 data points, and the minute-to-minute data are from 1998 to 2003 with 50 064 data points. The minute-to-minute data are recorded every minute in the German market, while every 5 min in the Chinese market. A working day is about 450 min in Germany while exactly 240 min in China. In our terminology, the results of the "Chinese Indices" are the averages of the Shanghai Index and Shenzhen Index.

Denoting a financial index at time t as $P(t)$, the return and volatility are defined as $R(t) \equiv \ln P(t+1) - \ln P(t)$ and $|R(t)|$ respectively. Naturally, the dynamic properties of volatilities may depend on the time scale. To study the dynamic relaxation after and before the large fluctuations, we introduce the remnant and anti-remnant volatilities,

$$v_{\pm}(t) = \left[\langle |R(t' \pm t)| \rangle_c - \sigma\right]/Z, \tag{1}$$

where $Z = \langle |R(t')| \rangle_c - \sigma$, σ is the average volatility, and $\langle \cdots \rangle_c$ represents the average over those t' with specified large volatilities. In our analysis, the large volatilities are selected by the condition $|R(t')| > \zeta$, and the threshold ζ is well above σ, e.g., $\zeta = 2\sigma$, 4σ, 6σ, and 8σ. At the daily time scale, when ζ is sufficiently large, the selected events correspond to the financial crashes or rallies. At the time scale in minutes, the events may be only extremal fluctuations. The remnant volatility $v_+(t)$ describes how the system relaxes from a large fluctuation to the stationary state, while the anti-remnant volatility $v_-(t)$ depicts how it approaches a large fluctuation.

Large shocks in volatilities are usually followed by a series of aftershocks. Thus we *assume* that both $v_+(t)$ and $v_-(t)$ obey a power law,

$$v_{\pm}(t) \sim (t + \tau_{\pm})^{-p_{\pm}}, \tag{2}$$

where $p_{\pm}$ are the exponents and $\tau_{\pm}$ are positive constants. In most cases studied in this paper, the constants $\tau_{\pm}$ are rather small. For reducing the fluctuations, we integrate Eq. (2) from 0 to t. Thus the cumulative function of $v_{\pm}(t)$ is written as

$$V_{\pm}(t) \sim \left[(t + \tau_{\pm})^{1-p_{\pm}} - \tau_{\pm}^{1-p_{\pm}}\right] \tag{3}$$

for $p_{\pm} \neq 1$. The power-law behavior of $v_{\pm}(t)$ just represents the long-range temporal correlation of volatilities. Such a power-law behavior has been well understood in dynamic critical phenomena, even in the case far from equilibrium [20,21].

Our main motivation in this paper is to explore different characteristics of the large-fluctuation dynamics at the daily time scale and the time scale in minutes, especially the time-reversal symmetry or asymmetry. We first analyze the minute-to-minute data of the Chinese Indices and German DAX. Now $|R(t)|$ is calculated in units of five minutes for the Chinese Indices and one minute for the German DAX. For the minute-to-minute data, a large volatility may not indicate a real macroscopic crash or rally, and it only possibly brings the dynamic system to a *microscopic* non-stationary state. In Fig. 1(a), $V_{\pm}(t)$ of the Chinese Indices are displayed on a log–log scale. Due to the so-called intra-day pattern [22,23,3,8], the curves periodically fluctuate at a working day, i.e., $t \sim 240$ minutes. Such a kind of intra-day patterns should be removed. Following the procedure in Refs. [3,24], the intra-day pattern $D(t'_{day})$ is defined as

$$D(t'_{day}) = \frac{1}{N} \sum_{j=1}^{N} |R_j(t'_{day})|, \tag{4}$$

where j runs over all the trading days N, and t'_{day} is the time in a trading day. To remove this intra-day pattern, we normalize the volatility at time $t' = t'_{day}$ by

$$r(t'_{day}) = |R(t'_{day})|/D(t'_{day}). \tag{5}$$