Physica A 392 (2013) 5431–5452

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Environmental superstatistics

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HIGHLIGHTS

- First application of superstatistics to temperature distributions relevant in environmental physics.
- Long-term distributions have double peak.
- Global warming visible in data.
- Practical relevance for thermodynamic devices working outdoors.

ARTICLE INFO

Article history:
Received 23 December 2012
Received in revised form 24 May 2013
Available online 29 June 2013

Keywords:
Superstatistics
Surface temperature distributions
Global warming
Köppen–Geiger climate classification

ABSTRACT

A thermodynamic device placed outdoors, or a local ecosystem, is subject to a variety of different temperatures given by short-term (daily) and long-term (seasonal) variations. In the long term a superstatistical description makes sense, with a suitable distribution function $f(\beta)$ of inverse temperature β over which ordinary statistical mechanics is averaged. We show that $f(\beta)$ is very different at different geographic locations, and typically exhibits a double-peak structure for long-term data. For some of our data sets we also find a systematic drift due to global warming. For a simple superstatistical model system we show that the response to global warming is stronger if temperature fluctuations are taken into account.

1. Introduction

In nonequilibrium statistical mechanics, the superstatistics technique [1] is a powerful tool to describe a large variety of complex systems for which there is a change of environmental conditions and temperature fluctuations on a large scale [2–10]. A superstatistical complex system is mathematically described as a superposition of several statistics, one corresponding to local equilibrium statistical mechanics and the other one corresponding to a slowly varying parameter β of the system. Essential for this approach is the fact that there is sufficient time scale separation, i.e. the local relaxation time of the system must be much shorter than the typical time scale on which β changes. Many interesting applications of the superstatistics concept have been worked out for a variety of complex systems, for example the analysis of train delay statistics [11], hydrodynamic turbulence [12], cancer survival statistics [13] and some other applications as well, see Refs. [14–19].

In this paper we want to deal with environmental aspects of superstatistics, in the sense that we ask what distributions of inverse temperatures are seen by thermodynamic devices that are kept in open air outside a constant-temperature environment. Clearly this question is technically very relevant as many devices need to operate under strong temperature fluctuations, as given by either daily temperature fluctuations, or seasonal variations, or even long-term climatic changes.

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http://dx.doi.org/10.1016/j.physa.2013.06.057

Besides thermodynamic devices, one may also be interested in local complex ecosystems (such as e.g. biological populations) coupled to a changing temperature environment. We will analyse in detail inverse temperature distributions at various geographic locations. These environmentally important distributions are different from standard examples of distribution functions discussed so far in the literature, such as the χ-square, inverse χ-square or lognormal distribution [2]. As it is well known, superstatistics based on χ-square distributions lead to q-statistics [20,21], whereas other distributions lead to something more complicated. A major difference is that the environmentally observed probability densities of inverse temperature typically exhibit a double-peak structure, thus requiring a different type of superstatistics than what has been done so far. In the original setting described in Ref. [1], the analysis was based on single-peaked distributions with a sharp maximum [1]. For environmentally relevant temperature distributions, this concept has to be broadened.

A central point for the applicability of the superstatistics concept is the existence of suitable time scale separation of the dynamical evolution, or more generally the existence of a hierarchy of time scales which are well separated. In the simplest case this just means there are two different time scales such that the typical variation of β takes place on a much larger time scale than the local relaxation time of the system that is influenced by the given temperature environment. For meteorological and climatic systems there is indeed a hierarchy of different relevant time scales. It starts with time scales due to the turbulent dynamics of the air which are well below the daily temperature oscillations; also there are synoptic meteorological disturbances of the order of about 2 days. We arrive at seasonal variations (circular statistics) and at larger scales at inter-annual variability. Finally, at the largest scales there are long-term trends due to climatic and geological changes. Our aim in this paper is not a detailed analysis of all these various dynamical phenomena on various time scales (see, e.g. Refs. [22–26] and references therein), but the application of superstatistical techniques, given the fact that there is time scale separation. This means effectively we consider a given subsystem or device (which may be a technical thermodynamic device, but in a more general setting also a given local ecosystem depending on temperature and precipitation dynamics) and then look at a generalized statistical mechanics description of it. For this it is necessary to fully understand the statistics of (inverse) temperature fluctuations—either as sampled over long-term records or conditioned on particular periods (say summer or winter). While in principle one could also do superstatistics on shorter (turbulent) time scales of air movement, in this paper we restrict ourselves to the long-term statistical properties.

This paper is organized as follows. In Section 2 we look at monthly data (essentially eliminating seasonal variations) at various geographical locations, and check how well the data are described by Gaussian distributions. In Section 3 we look at long-term data including seasonal variations, which induce double-peaked distributions, but with specific differences at different geographical locations, depending on local climate. In Section 4 the results are interpreted in terms of the Köppen–Geiger climate classification scheme. In Section 5 we discuss why the superstatistics relevant for environmental temperature fluctuations is different from what has been done so far for single-peaked distributions. In Section 6 we deal with global warming, and the interesting effects that occur due to fluctuations in superstatistical models if a parameter such as the mean of the temperature distributions increases slightly. Finally, our conclusions are summarized in Section 7.

2. Observed inverse temperature distributions—monthly data

When looking of temperature distributions as given by real data, one clearly has to specify the relevant time scale first. Short-term temperature distributions (dominated by daily fluctuations) are very different from long-term data (dominated by seasonal variations). For very long data records, one also has to take into account non-stationary behaviour due to climate change.

We start with short-term data. Figs. 1–3 show as an example a time series of hourly measured surface inverse temperatures in Ottawa during November 2011, as well as in Vancouver during May 2011 and December 2011.

The frequency pattern is dominated by 1 oscillation per 24 h, corresponding to the day–night temperature differences. Superimposed to this are stochastic fluctuations due to different weather conditions and turbulent fluctuations. What is also clearly visible in the figures are small systematic trends. In Fig. 1 (November in Ottawa) the average inverse temperature β is slightly increasing over the month of November, since the temperature slightly decreases as winter is approaching. In Fig. 2 (May in Vancouver) the average β is slightly decreasing, since summer is approaching. No clear systematic trend is visible for the December data of Fig. 3.

The (inverse) temperature time series for a single month is approximately Gaussian distributed. This is illustrated in Figs. 4–6, which shows the histograms of the example time series of Figs. 1–3.

Consider now a thermodynamic device with energy levels E_i that is kept outside and hence subject to slowly varying inverse temperatures. Our interpretation of 'thermodynamic device' is very general here. One may also think of an ecosystem or other complex system (including biological populations) that is influenced or in fact, dependent, on the temperature (and precipitation) of its environment. Suppose that locally, for constant inverse temperature, the device is properly described by statistical mechanics with ordinary Boltzmann factors $e^{-\beta E}$. Assuming time scale separation, so that the system can quickly enough relax to local equilibrium, the long-term behaviour is properly described by a mixture of Boltzmann factors with different temperatures, weighted with a function $f(\beta)$ that describes how often a given inverse temperature of the environment is observed. This is the realm of superstatistics [1]. The effective statistical mechanics of this thermodynamic device (or ecosystem) is indeed described by effective Boltzmann factors $B(E)$ of the form

$$B(E) = \int_0^\infty f(\beta) e^{-\beta E} d\beta \tag{1}$$