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Non-monotonic increase of robustness with capacity tolerance in power grids

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HIGHLIGHTS

- The robustness of a power grid does not always increase monotonically with capacity.
- The cascading failure is divided into several sub-stages.
- The number of overloaded nodes and the remaining load in each sub-stage are analyzed.
- Increasing capacity just reduces the number of overloaded nodes in the sub-stage 1.
- The reasons for the non-monotonic variations of power grid robustness are given.

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ABSTRACT

The robustness of different scale power grids is analyzed based on complex network theory in terms of electrical betweenness and weighted efficiency. The robustness of a power grid does not always increase monotonically with the capacity. This property is different from the results obtained in previous studies, which have indicated that the robustness increases monotonically with capacity. To understand the non-monotonic phenomenon, the cascading failure is divided into several sub-stages, and we analyze the number of overloaded nodes and the average remaining load in each sub-stage. The results indicate that the increasing capacity is barely able to reduce the number of overloaded nodes at the beginning of malfunction, which may lead to more nodes being removed subsequently, including certain nodes with many connections or large load. More loads remain in the power grid such that certain nodes cannot take the load. This eventually causes overloading of more nodes and a decline in the robustness of the power grid. The conclusion may be useful for power grid planners seeking to design grids with cost-effective capacity.

1. Introduction

Currently, large interconnected power grids, as one of the most complex types of networks, are facing the challenge of vulnerability. Several well-publicized blackouts have occurred and resulted in catastrophe, such as the Western North American blackouts in 1996 [1], the blackouts in North America and London in August 2003 [2,3] and the European grid power failures in 2006 [4]. To understand the mechanism of these blackouts, two types of approaches have been proposed based on the complex network theory. The first approach involves static failures. That is, a certain proportion of elements are continuously removed to measure the robustness of network performance. Albert [5], Bompard [6,7],

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Wang [8] and Holm [9] adopted this approach to study the vulnerability of homogeneous and heterogeneous networks. These researchers' results indicated that homogeneous networks are robust against both random failures and deliberate attacks, whereas heterogeneous networks are robust against random failures but fragile to deliberate attacks (the robust-yet-fragile property). Buldyrev [10] and Huang [11,12] presented a mathematical framework and studied the robustness of interdependent networks using the static approach. These authors' research findings indicate that interdependent networks are more vulnerable to failures than single networks. The second approach is the dynamical approach, which considers the redistribution of flows when a single or a small proportion of elements is removed. Using this approach, Motter [13], Wang [14–17], Yu [18], Crucitti [19,20], Kinney [21], Chen [22], Ding [23] and Wang [24] also observed the robust-yet-fragile property of heterogeneous networks. To improve the robustness of heterogeneous networks against deliberate attacks, Motter [25] intentionally removed some nodes (having small load) and edges (having large excess of load) immediately after the initial attack. The robustness of the heterogeneous networks increased drastically.

From these previous works, it can be concluded that higher capacity is helpful for improving the robustness of complex networks (i.e., the robustness monotonically increases with the capacity). However, in the present work, we found that the increase of capacity is not necessarily helpful for enhancing network robustness (i.e., the robustness non-monotonically varies with the capacity). Specifically, we used the electrical betweenness as the initial load defined by Wang [8], redefined the efficiency of the network based on the work of Latora [26] and analyzed the robustness change of a power grid with capacity. In contrast from the previous conclusions, the simulation results indicate that the robustness of the power grid has a non-monotonic increase when capacity increases occasionally. This phenomenon has been shown in Ref. [27] (see Figs. 2 and 4); however, no detailed analyses and explanations has been performed. Here, we have divided the cascading failures into several sub-stages and have analyzed the number of overloaded nodes and the average remaining load in each sub-stage. Furthermore, the reasons for the non-monotonic variations of network robustness are given.

The rest of this paper is organized as follows. Section 2 introduces the cascading model in detail. The measurements of network robustness are presented in Section 3. In Section 4, the robustness of the IEEE-39, IEEE-57, IEEE-118, IEEE-145 and IEEE-162 bus power system is investigated, and the reasons for the non-monotonic increase phenomenon are analyzed. We also analyze the statistical characteristics in Section 5. The conclusions are presented in Section 6.

2. Cascading failure model for power grid

In complex networks theory, a power grid can be abstracted as a weighted graph $G(N, K)$ [26,28], with N nodes (the generation substations G_G, the transmission substations G_T and the distribution substations G_D) and K edges (the transmission lines). G is represented by an $N \times N$ adjacency matrix $\{y_{ij}\}$, where the element y_{ij} is 0 if there is no direct line from the substation i to the substation j. Otherwise, y_{ij} is the admittance of edge ij.

Initially, a power grid functions properly if the load of each node is within its capacity. The capacity of node k is defined as

$$C_k = T \times L_k(0) \tag{1}$$

where T is the tolerance parameter, generally speaking $T \geq 1$. $L_k(0)$ is the initial load of node k. The removal of some nodes will break the balance of the load and result in load redistribution over remaining nodes. Occasionally, this can further trigger some nodes overloading and new load redistribution. This cascading process will not stop until no nodes overload.

In a power grid, there is a common Kirchhoff's law for all elements to follow, and thus, the electrical betweenness [8] is used as the initial load. Generally, the electrical betweenness of node k can be given as the following:

$$B(k) = \begin{cases} \left(\sum\limits_{l \in F(k)} B(kl) + \sum\limits_{j \in G_D} w_{kj} \right) \Big/ 2, & k \in G_G \\ \left(\sum\limits_{l \in F(k)} B(kl) + \sum\limits_{j \in G_G} w_{jk} \right) \Big/ 2, & k \in G_D \\ \left(\sum\limits_{l \in F(k)} B(kl) \right) \Big/ 2, & k \notin G_G \text{ and } k \notin G_D \end{cases} \tag{2}$$

where $w_{kj} = \min(S_k, S_j)$, S_k is the nominal output of generation substation k, and S_j is the maximal demand of distribution substation j. G_G is the set of generation nodes, and G_D is the set of distribution nodes. $F(k)$ is the node set connecting to node k. $B(kl)$ is the electrical betweenness of edge kl, which is defined as follows:

$$B(kl) = \sum_{i \in G_G, j \in G_D} w_{ij} \, |I_{kl}(i, j)| \tag{3}$$

$$I_{kl}(i, j) = y_{kl}[(Z_{ik} - Z_{jk}) - (Z_{il} - Z_{jl})] \tag{4}$$

where y_{kl} is the admittance of edge kl, and Z_{ik}, Z_{jk}, Z_{il} and Z_{jl} are the relevant elements of the power grid impedance matrix.