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Multifractal analysis of stock exchange crashes

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ABSTRACT

We analyze the complexity of rare events of the DJIA Index. We reveal that the returns of the time series exhibit strong multifractal properties meaning that temporal correlations play a substantial role. The effect of major stock market crashes can be best illustrated by the comparison of the multifractal spectra of the time series before and after the crash. Aftershock periods compared to foreshock periods exhibit richer and more complex dynamics. Compared to an average crash, calculated by taking into account the larger 5 crashes of the DJIA Index, the 1929 event exhibits significantly more increase in multifractality than the 1987 crisis.

1. Introduction

It is widely accepted that financial markets illustrate strong signs of complex dynamical systems and the distribution of returns of high frequency data follows a power law. In this context, financial time series exhibit non-linear properties and the stylized facts call for long-memory, fat tails and multifractality [1,2]. Particularly for stock exchange time series, fat tails, power-law correlations and multifractality have been documented in a number of cases [3–5]. These results are in disagreement with the traditional economic notion which states that markets act in accordance with the Efficient Market Hypothesis (EMH).

The majority of the empirical financial studies aimed in identifying long term correlations either in single or multiple time series data [6–10]. Unlike large and intraday time series data examination, extreme events of stock exchanges have received little attention. In our situation we are interested in investigating the statistical properties of stock exchange indexes during periods of high stress and namely periods of stock market crashes with emphasis given to 1929 and 1987 crashes. Ref. [11] analyzed similar extreme event impact but on the exchange rate markets, while [9] for 88 companies that contribute to the S&P 500 index, during the 26-year period 1983–2009, apply time-lag Random Matrix Theory (TLRMT) for each year and show pronounced peaks in TLRMT singular values during the largest market shocks and economic crises: Black Monday, the Dot-com bubble and the 2008 crash.

The purpose of investigating market crashes is based on scientific evidence that such complex systems reveal their structure better when they are under stress than in normal conditions. According to Sornette, [12] "such extreme events express more than anything else the underlying 'forces' usually hidden by almost perfect balance and thus provide the potential for a better scientific understanding of complex systems". Consequently the examination of these specific great crashes will provide an understanding about the dynamics and complexity of the stock exchange markets will assist especially institutional investors to correctly assessed market risk and, also, it will provide to the policy makers the information needed to put in place the appropriate mechanism in encountering future problems.

In this vein, the work of Refs. [13,14], using intraday data, report a correlation between the width of the estimated multifractal spectrum and future price fluctuation, putting the basic for price fluctuations prediction and thus to future

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crashes. But Ref. [15] after testing the above statement, with rigorous research, has come to the conclusion that the multifractal nature in the indexes is not a fact but fiction. This result was further supported by analyzing two additional indexes (S&P 500 and NASDAQ) in developed stock markets. Furthermore Ref. [16], suggest that it is valuable to apply the partition function approach to the multifractal analysis of stocks and indexes and to the possible application of multifractal properties in market forecasting and managing risk, but by using returns series rather than stock prices or indexes.

In this paper, we first calculate the market complexity of the two crises and later we divide the total sample into periods before (foreshocks) and after (aftershocks) the crash in an attempt to identify the changes (if any) in the market dynamics and complexity. In addition we analyze three other main crashes and then we calculate the average crash based on all these rare Dow Jones Industrial Average (DJIA) market events. Last we present the generalized Hurst exponent and we provide some concluding remarks.

2. Methodology

We examine the nonlinear features of extreme events of DJIA, starting with the 1929 and 1987 crashes. The data consist of daily returns of the stock market index before and after the market crash and spans from August 1928 to January 1931 and August 1986 to December 1988 respectively. We are interested in investigating the whole process of the event, i.e. energy accumulation, or bubble rising and the expansion process after the eruption. Therefore the data consist of around three years of trading days, sufficient enough in revealing the statistical properties and relevant information. The dates of the two crashes were Oct. 28, 1929 with the DJIA losing 14.5% of its value and Oct. 19, 1987 with the loss amounting to 25.6%. The daily rate of returns of the stock market is calculated as follows:

$$r_t = \ln p_t - \ln p_{t-1} \tag{1}$$

where $p(t)$ is the price of the index on day t and r is the rate of return.

We are investigating the multifractal properties of the above episodes. Numerous procedures have been produced in calculating multifractality. In our case we use the Multifractal-Detrended Fluctuation Analysis (MF-DFA) developed by Ref. [17] as this method reduces noise effects, removes local trends and avoids spurious detection of correlations that are artifacts of nonstationarities in the time series. Motivated by DFA, a detrended cross-correlation analysis (DCCA) was introduced [18,19] together with its multifractal extension [20,21]. It was also shown that for short time series, like in our case, and negative moments, the significance of results for MF-DFA is better than most of the other techniques. The multifractal generalization of the MF-DFA procedure can be briefly sketched as follows. The MF-DFA operates on the time series $x(k)$, where $k = 1, 2, \ldots, N$ and N is the length of the series. We assume that $x(k)$ are increments of a random walk process around the average $\langle x \rangle$ and the profile is given by the integration of the signal

$$Y(i) = \sum_{k=1}^{N} [x(k) - \langle x \rangle], \quad i = 1, \ldots, N. \tag{2}$$

Next, the time series $Y(i)$ is divided into $N_S \equiv \text{int}(N/s)$ non-overlapping segments of equal length s, starting from both beginning and the end of the time series. Each segment υ has its own local trend that can be approximated by the least-squares fitting of the series. Then we determine the variance

$$F^2(\upsilon, s) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[(\upsilon - 1)s + 1] - y_\upsilon(i)\}^2 \tag{3}$$

for each segment ν, $\nu = 1, \ldots, N_S$ and

$$F^2(\upsilon, s) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[N - (\upsilon - N_s)s + 1] - y_\upsilon(i)\}^2 \tag{4}$$

for $\nu = N_S + 1, \ldots, 2N_S$. Here, $y_\nu(i)$ is the fitting line in segment ν. Then, we detrend the series and average over all segments to obtain the qth order fluctuation function

$$F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{\upsilon=1}^{2N_s} [F^2(\upsilon, s)]^{q/2} \right\}^{1/q}. \tag{5}$$

The property of $F_q(s)$ is that for a signal with fractal properties, it reveals power-law scaling within a significant range of s

$$F_q(s) \propto s^{h(q)} \tag{6}$$

and the variable q can take any real value other than zero.

In general the exponent $h(q)$ will depend on q. For stationary time series, $h(2)$ is the well-defined Hurst exponent H and thus, $h(2)$ is the generalized Hurst exponent. Multifractal (MF) scaling exponent $\tau(q)$ is related to $h(q)$ through

$$\tau(q) = qh(q) - D_f \tag{7}$$