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Mixed boundary states from 1-loop bosonic closed strings

R. Nardi

Centro Brasileiro de Pesquisas Físicas - CBPF, R. Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro - RJ, Brazil

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ABSTRACT

We analyze in a simple calculation the way 2-dimensional moduli induce a noise-like mixing between the right and left sectors of the 1-loop partition function of bosonic closed strings. As it is a direct consequence of string interactions, we argue it must be taken as a renormalization to the tree-level dynamics as far as the Wilsonian view on renormalization group is kept. We also highlight the way this noise effect brings a thermodynamical meaning to space-filling *D*-branes.

1. Introduction

It is fair to say the Wilsonian view on the renormalization group (RG) [1] constitutes one of the biggest achievements physicists have gotten in the last quarter of the 20th century.

Besides the fact that it has provided a consistent physical interpretation of the undesired divergences present in the old renormalization procedures through a downgrading of quantum field theories to merely the dominant dynamics in a specific point of the energy scale, it has also put in evidence conformal transformations as a device that connects the physics of different points in such a scale.

As a result, conformal symmetry appears in a special position when compared to the other symmetries, that is to say, its presence signals a *standing still* situation of the whole dynamics under the conformal transformations and not just an equivalence between some of the degrees of freedom.

In four dimensions a particular property simplifies the checking of conformal invariance: it holds whenever dilatations holds. In QCD, this fact, when added to the asymptotic freedom as well as to the running of masses to zero in high momenta signals that in fact one might expect conformal symmetry to be a very important piece of the extremal high energy physics.

The investigation of how conformal symmetry enters into the game in high energy physics is much enriched in string theory since, as is well known, a conformal group in 2 dimensions has infinite generators. In spite of its beauty, the full stringy physics would then be insensitive to any scale and therefore leaves unanswered the important question of how string theory is connected with the low-energy quantum field theories.

In this direction, an important observation is the fact that the hypothesis of a physical cut-off, present in the Wilsonian RG, actually hides the assumption of our lack of information about what is going on at the more microscopic layers of nature, i.e., one is actually facing an information/statistical/thermal problem [2].

In the context of thermal physics, one may say that temperature and renormalization are tied together, since through the second law of thermodynamics

$$\Delta E = T\Delta S, \tag{1}$$

a change in the energy level's amplitudes (i.e., a change in the norm of states) corresponds to a change in the average energy which induces a change in entropy times the temperature. From an information theory perspective such a connection may

E-mail address: rnardi@cbpf.br.

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also be seen in the presence of an unknown sector of degrees of freedom (thermal reservoir). Both points of view suggest that a thermal system shall share features with a set of renormalized degrees of freedom.

Back to the context of string theory, as strings has been seen as the most promising candidate for a microscopic description of gravity, it is the author's belief that pursuing a deep investigation about the thermodynamical meaning of conformal transformations in string theory might be worth doing as far as one takes into account the inherent thermodynamical content of gravity as signaled in Hawking–Bekenstein entropy of black holes and, in particular, the more recent stringy-inspired perspectives on the gravitational interaction [3,4] which, to some extent, push forward a whole line of research exploring gravity as a macroscopic effect [5–12].

In this sense it can be expected that the connection of string theory to the low-energy QFTs might involve some kind of (thermal) noise interaction among the string states giving rise to condensates, once noise, in general, means a statistical mixture. In other words, it is an open question the issue of what is the effective dynamics of strings at low energies.

Despite the fact that investigations about the relation between world-sheet and space–time renormalization groups were carried out [13] in the early 1990s, it seems that not much attention has been given to it since the discovery of *D*-branes. The point is indeed worth exploring provided the breaking of target-space Poincaré symmetry (and therefore breaking of target-space conformal symmetry) must have a counterpart from a world-sheet perspective.

In this letter we push forward the investigation of how the energy scale integration in bosonic string theory which is accomplished by the world-sheet moduli hides a noise integration among pure states.

The major result is that the noise we have found arises in coherent states which in a specific limit realize the boundary states associated to *D*-branes. The noise-free state matches the *D*-brane state. The result is in agreement with the expected picture of emergence of *D*-branes as condensed states of closed strings at low energies. The calculation goes along with the picture of effective dynamics of strings at low energies where, among other things, conformal invariance no longer holds.

2. 1-loop interactions of bosonic strings

In perturbative bosonic strings scattering amplitudes are calculated taking into account the topological contributions coming from the Euler characteristic

$$\chi\left(\Sigma\right)=\frac{1}{4\pi}\int_{\Sigma}\mathrm{d}^{2}\sigma\sqrt{g}R=2\left(1-g\right) \tag{2}$$

where g is the genus of Σ. The Euler characteristic $\chi\left(\Sigma\right)$ provides a loop-counter for the quantum string making it possible to weight a functional integral with an interacting dynamics. The resulting (covariantly gauge-fixed) Wick-rotated action is then

$$S=\frac{1}{4\pi\alpha'}\int_{\Sigma}\mathrm{d}^{2}\sigma\sqrt{\hat{g}}\hat{g}^{\alpha\beta}\partial_{\alpha}X^{\mu}\partial_{\beta}X_{\mu}+\frac{1}{2\pi}\int_{\Sigma}\mathrm{d}^{2}\sigma\sqrt{\hat{g}}b^{\alpha\beta}\partial_{\alpha}c_{\beta}+\chi\left(\Sigma\right), \tag{3}$$

where the hat stands for the gauge-fixed world-sheet metric.

Handles in Σ are related to the way the topological compactification (of the world-sheet) is performed and induce globally non-trivial 2-dimensional background metric configurations, the *moduli*, which realize zero-modes of the adjoint ghost wave operator being therefore irreducible to the identity metric through gauge transformations (Diff + Weyl).

At 1-loop ($\chi\left(\Sigma\right)=0$) one deals with the topology of a torus whose compactification (modulo global Weyl transformations) is parameterized by a length τ_2 and an overall "*torsion*" angle τ_1. Modular transformations fix the fundamental region of the Teichmüller (modulus) parameter $\tau=\tau_1+i\tau_2$ to be $-1/2<\tau_1<1/2$ and $|\tau|>1$. Provided one fixes the world-sheet volume to be 1, τ enters into the global 2d-metric as follows

$$g_{(\tau)}=\frac{1}{\tau_{2}}\begin{pmatrix}1 & \tau_{1}\\ \tau_{1} & |\tau|^{2}\end{pmatrix}. \tag{4}$$

Since each value of τ defines an equivalence class under gauge transformations, a properly defined functional integral must consider the whole set of equivalence classes while defining the quantum theory. On the other hand, as τ_2 measures the torus length, it works as a correlation length and therefore has an energy scale associated to it. In other words, integration over moduli-space is in fact an energy scale integration, i.e. RG integration.

Taking this into account we pursue the natural question of what kind of quantum state at each level of the energy scale does contribute to the bosonic string partition function at 1-loop.

We use the Fourier decomposition of the bosonic string

$$X^{\mu}\left(\sigma^{1},\sigma^{2}\right)=x^{\mu}-\mathrm{i}\alpha'p^{\mu}\sigma^{2}+\mathrm{i}\sqrt{\frac{\alpha'}{2}}\sum_{n\neq0}\left\{\frac{\boldsymbol{\alpha}_{n}^{\mu}}{n}\mathrm{e}^{-n\left(\mathrm{i}\sigma^{1}+\sigma^{2}\right)}+\frac{\bar{\boldsymbol{\alpha}}_{n}^{\mu}}{n}\mathrm{e}^{-n\left(\mathrm{i}\sigma^{1}-\sigma^{2}\right)}\right\} \tag{5}$$

and consider the Polyakov action defined with $g_{(\tau)}$

$$S_{P_{(\tau)}}=\frac{-1}{4\pi\alpha'}\int_{\Sigma}\mathrm{d}^{2}\sigma\sqrt{g_{(\tau)}}g_{(\tau)}^{\alpha\beta}\partial_{\alpha}X^{\mu}\partial_{\beta}X_{\mu}. \tag{6}$$