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# Position space renormalization group study of the spin-1 random semi-infinite Blume–Capel model

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ABSTRACT

We study the spin-1 Blume–Capel model under a random crystal field in the tridimensional semi-infinite case. This has been done by using the real-space renormalization group approximation and specifically the Migdal–Kadanoff technique. Interesting results are obtained, which tell us that the randomness destroys the first order phase transitions and only those of the second order occur. We give the list of nine fixed points and their topology describing the surface critical behavior. Five new types of phase diagram are found with a rich variety of phase transitions, in accordance with the values of the bulk and surface probabilities and the ratios linking bulk and surface interactions.



## 1. Introduction

The Blume–Capel model has been the subject of numerous studies and has led to many interesting developments [1,2]. It is a spin-1 Ising model with a single anisotropic ion, which was first introduced to study the first order magnetic phase transitions; it was thereafter used to study multicomponent fluids [3,4]. The Blume–Emery–Griffiths model [5] was introduced later as a generalization of the Blume–Capel model; it was applied to investigate isotopic mixtures of helium $^3$He–$^4$He and multiple physical systems. Several approximations methods (the mean field approximation [1,2,6], effective field theory [7], techniques of the renormalization group [8,9] and Monte Carlo simulation [10]) have been employed to study the Blume–Capel model.

On the other hand, the Blume–Capel model has been studied on semi-infinite systems to highlight the surface effects on the phase transitions [11–13]. Since the neighborhoods of microscopic interactions at the surface can be quite different from those in the bulk, the critical behavior of the surface can present certain complexity. Thus, it is possible that the surface system orders before the bulk or one can assist at apparition of order phenomena of different features to those of the bulk [14], when the ratio between the bulk and surface interactions exceeds a certain critical value. In such a situation (we refer in particular to Refs. [11,12]), we talk about surface and extraordinary phase transitions where the temperature of the surface transition is higher than that of the extraordinary transition. Another situation can occur when the order is established in the bulk and on the surface at the same temperature, which is known as an ordinary transition. However, when the system transition takes place between the two previous situations we talk about a special transition, in other words when the extraordinary and surface transitions appear simultaneously.

Furthermore, numerous studies have been reported in order to understand the effect of random fields [15,16] on physical systems. These have been done by using several models, including that of Blume and Capel. The presence of randomness

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produces remarkable impacts on the critical behavior of the systems, by removing the first order phase transitions totally or partially and leaving the second order ones [17,18]. The existence of a critical probability [19] allows one to distinguish between different main types of phase diagram with different behavior. Let us note that the exact form of the probability distribution does not play a fundamental role; only the presence of randomness is important.

In our present work, we bring many contributions. First, we study the spin-1 Blume–Capel model with a random crystal field on a hypercubic lattice in the infinite case as well as in the semi-infinite one by using the techniques of the renormalization group, more precisely that of Migdal and Kadanoff [20,21]. But one of the main difficulties in studying this system lies in the estimation of the renormalized probability and crystal field. Indeed, because of the non-constancy of the sign of the crystal field peaks, direct application of the Stinchcombe–Watson approximation [22] gives aberrant values for the renormalized probability. We show how to transcend this difficulty by estimating at each iteration the values of the different peaks and by doing the sharing between the positive and negative peaks, which allows us to find coherent values of the renormalized probability. This correction is of great use in establishing the recursion equations of the random infinite and semi-infinite models. In the infinite random model, we recover two types of phase diagram, separated by a critical probability. This finding is common for two-dimensional and three-dimensional cases. Another main contribution of our study is that we establish five types of phase diagram in the semi-infinite random Blume–Capel model, which allows a better understanding of the surface effects. In these diagrams, we observe surface, ordinary, extraordinary and special phase transitions, in accordance with the values of the bulk and surface probabilities and the ratios linking bulk and surface interactions. These transitions belong to four different universality classes. We find that the presence of randomness destroys the first order phase transitions, leaving only those of second order.

Our paper is organized as follows. In Section 2, we treat the infinite random Blume–Capel model, in which the Migdal–Kadanoff renormalization group technique will be applied to determine the recursion equations and to study the system critical behavior. The semi-infinite random Blume–Capel model will be the subject of Section 3, where interesting results will be reported and different discussions will be given about the fixed points and phase diagrams found. Finally, we give our conclusion in Section 4.

## 2. Infinite random Blume–Capel model

To treat the infinite random case, we consider the spin-1 Blume–Capel model. This is described by the Hamiltonian

$$-\beta H = J\sum_{i,j} S_i S_j - \sum_i \Delta_i S_i^2, \tag{1}$$

where $S_i = 0, \pm 1$, $J$ is the reduced bilinear interaction and $\Delta_i$ is the crystal field at site $i$. The first summation runs over all first nearest neighbor pairs of the lattice and the second one over all sites.

Although the reduced biquadratic interaction $K$ is equal to zero in the Blume–Capel model, we will take it into account because we are using renormalization group techniques that need its presence in the Hamiltonian. Indeed, the subspace $K = 0$ is not invariant by renormalization for the Blume–Emery–Griffiths model defined as

$$-\beta H = J\sum_{i,j} S_i S_j + K\sum_{i,j} S_i^2 s_j^2 - \sum_i \Delta_i S_i^2, \tag{2}$$

where $J$ and $K$ are the coupling constants between two sites $i$ and $j$ of the $d$-dimensional lattice while $\Delta_i$ is a local interaction at each site, taking into account the vicinity of the considered site. The phases of the Blume–Capel model are defined by the values taken by two order parameters: the magnetization $m$ and the quadrupolar momentum $q$. Note that for an ordered phase $m = \langle S_i\rangle = 1$ and $q = \langle S_i^2\rangle = 1$, but for a disordered phase $m = 0$ and $q = 0$ or 1.

We introduce a random crystal field obeying a probability distribution $P(\Delta_i)$ with two peaks where $\Delta_i = +\Delta$ with probability $p$ or $\Delta_i = -\Delta$ with probability $1 - p$. Therefore, this probability distribution can be written as

$$P(\Delta_i) = p\delta(\Delta_i + \Delta) + (1-p)\,\delta(\Delta_i - \Delta). \tag{3}$$

We use an approximation of the position space renormalization group that is the Migdal–Kadanoff one. It combines decimation and bond shifting, and permits thereby a more reliable qualitative appreciation of the phase transitions characteristics. Let us consider a $d$-dimensional lattice on which our model is placed. The decoupling is performed under the $d$ directions of the lattice, and let us call $(X)$ the direction on which the decimation is carried out with a spatial rescaling factor $b = 3$. In order to apply the renormalization procedure, we use the following chain (Fig. 1) with four spins, $S_1$, $S_2$, $S_3$ and $S_4$.

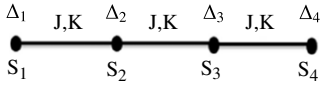


**Fig. 1.** Chain of four spins, $S_1$, $S_2$, $S_3$ and $S_4$. $J$ and $K$ denote, respectively, the reduced bilinear and biquadratic interactions, while $\Delta_i$ is the crystal field at site $i$. Decimation will be performed on the two middle spins, $S_2$ and $S_3$.