Physica A 392 (2013) 763–772

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The influences of delay time on the stability of a market model with stochastic volatility☆

Jiang-Cheng Li, Dong-Cheng Mei *

Department of Physics, Yunnan University, Kunming, 650091, China

ARTICLE INFO

Article history:
Received 10 June 2012
Received in revised form 11 September 2012
Available online 29 October 2012

Keywords:
Financial markets
Heston model
Time delay
Correlated noises
The mean escape time

ABSTRACT

The effects of the delay time on the stability of a market model are investigated, by using a modified Heston model with a cubic nonlinearity and cross-correlated noise sources. These results indicate that: (i) There is an optimal delay time τ_o which maximally enhances the stability of the stock price under strong demand elasticity of stock price, and maximally reduces the stability of the stock price under weak demand elasticity of stock price; (ii) The cross correlation coefficient of noises and the delay time play an opposite role on the stability for the case of the delay time $<\tau_o$ and the same role for the case of the delay time $>\tau_o$. Moreover, the probability density function of the escape time of stock price returns, the probability density function of the returns and the correlation function of the returns are compared with other literatures.

1. Introduction

It is common that stochastic processes are used to investigate the statistical properties of stock prices in finance [1–15]. The application of statistical physics to describe the behavior of financial markets has given rise to a new field called econophysics [1,2]. The most basic model is the geometric Brownian motion [3], but it cannot reproduce three important stylized facts observed in financial time series [2,4,10]: (i) The non Gaussian distribution of stock price returns; (ii) The fat tails; (iii) The stochastic character of volatility, which is characterized by long range memory and clustering. Therefore, many valuable models have been developed to picture the dynamics of the volatility, for instance, the ARCH [5], GARCH [6] and Heston [7] model, etc. In recent years, researchers have paid more attention to the Heston model. A Langevin equation approach to a model for stock market fluctuations and crashes is proposed based on an identification of the different processes influencing the demand and supply, and their mathematical transcription [8]. For example, an analytic formula for the time-dependent probability distribution of returns, exactly after integrating out the variance, is found by solving the corresponding Fokker–Planck equation [9]. The noise enhanced stability phenomenon (i.e., the noise induces an enhancement of the lifetime of a metastable state) is analyzed via the escape time in the Heston model [10]. A stabilizing effect of noise is also found after discussing the mean escape time in a modified Heston model with a cubic nonlinearity [11]. After studying the statistical properties of the hitting times in different models for stock market evolution, a noise enhanced stability phenomenon is found too [12]. The statistical properties of the escape times, or hitting times, for stock price returns by using different models which describe the stock market evolution are reported [13]. After solving the escape problem for the Heston random diffusion model, exact expressions for the survival probability (which amounts to solving the complete escape problem) as well as for the mean exit time are obtained [14], and an evidence of extreme deviations implies a high risk of default, as the strength of the volatility fluctuations increases [15].

☆ This work were supported by the National Natural Science Foundation of China (grant No. 11165016) and the program of IRTSTYN in Yunnan province.
* Corresponding author. Tel.: +86 08715032577; fax: +86 08715035570.
E-mail addresses: meidch@ynu.edu.cn, meidch@126.com (D.-C. Mei).

0378-4371/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2012.10.028

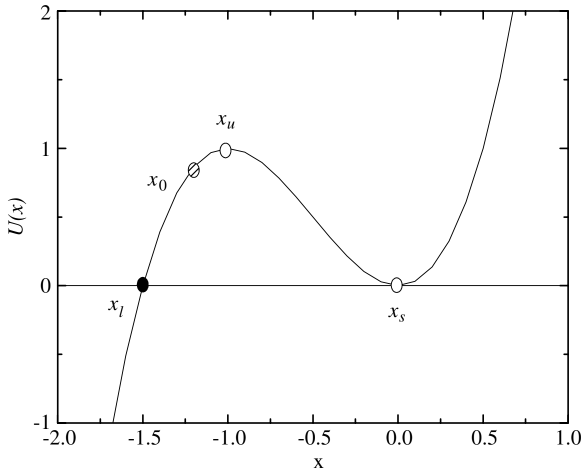

Fig. 1. Cubic potential used in the dynamical equation for the price $x(t)$. It indicates the starting positions used in our simulations.

However, in previous works, the effects of delay time have not been considered. In realistic systems, an inclusion of time delay is natural. Time delay is discovered in many dynamics systems, such as bistable systems [16], Brownian motors [17], biological systems [18–20], coupled chaotic maps [21], and so on. From the point of physics, the transport of matter, energy and information through the system requires some time which is treated as time delay. Homoplastically, there is delay time in the transmission of corporate news and governmental policies due to investors in different countries and regions and their different abilities of receiving and applying company news. The delay time can be found in some researches in financial markets. For example, finding delay time and using it for stock trend prediction based on a history of closing price of a number of high volatility stocks and consumer stocks [22]. The time delay block is found in the Trading Decision Maker [23]. A hybrid system with time delays based on neural networks and genetic algorithms for detecting temporal patterns in stock markets has been reported [24]. The cross-correlation exponents and crossovers demonstrating periodical uncertainty changing with the time-delay are found in US and Chinese stock markets by using the Detrended Fluctuation Analysis and Detrended Cross-Correlation Analysis methods [25]. It is found that applications of time-delayed evolution equations in these financial physics have been discussed as well [26–30]. Thus the financial market system with time delay needs to be investigated.

In a physical sense, the mean escape time of the Heston model is employed to describe the stability of stock price (SSP), representing the time of the stock price staying in a price range [10–12,14], but these researches have not considered the effects of delay time. In this paper, through numerically simulating the mean escape time of a modified Heston model, the effects of time delay on the stability of stock price are investigated with different demand elasticities of stock price (DESP), and the elasticity is the degree to which the curve of demand reacts to a change in the curve of the price [31]. Then the model evidencing its limits and features are discussed via comparing the probability density function (PDF) of the escape time of the returns, the PDF of the returns and the correlation function of the returns with those obtained from real financial time series of literatures [10–13]. In Section 2, a modified Heston model with an effective cubic potential is presented. Presenting the delay modified Heston model, the analyses of the mean escape time are given in Section 3. In Section 4 the statistical properties of stock price returns are discussed. In Section 5, a brief conclusion ends the paper.

2. The modified Heston model

Here after replacing the geometric Brownian motion with a random walk, a modified Heston model represents a Brownian particle moving in an effective potential with a unstable state, in order to model those systems with two different dynamical regimes like financial markets in normal activity and extreme days. The modified Heston model is described by the following system of coupled stochastic differential equations [8,10–12]

$$\begin{aligned}
\mathrm{d}x(t) &= -\left(\frac{\partial U}{\partial x} + \frac{\nu(t)}{2}\right)\cdot \mathrm{d}t + \sqrt{\nu(t)}\cdot \mathrm{d}Z(t),\\
\mathrm{d}\nu(t) &= a(b-\nu(t))\cdot \mathrm{d}t + c\sqrt{\nu(t)}\cdot \mathrm{d}W_c(t),\\
\mathrm{d}W_c(t) &= \lambda\cdot \mathrm{d}Z(t) + \sqrt{1-\lambda^2}\cdot \mathrm{d}W(t),
\end{aligned} \tag{1}$$

where $x(t)$ denotes the *log* of the stock price, the effective cubic potential U is $U(x) = px^3 + qx^2$ with $p = 2$ and $q = 3$ (see Fig. 1), $\nu(t)$ denotes the volatility of the stock price, a indicates the mean reversion of the $\nu(t)$, b indicates the long-run variance, c indicates the *volatility of volatility*, i.e., it is the amplitude of volatility fluctuations [32], $Z(t)$ and $W(t)$ are