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What if criminals optimize their choice? Optimal strategies to defend public goods

Camilla Bruni [a], Juan Carlos Nuño [b,*], Mario Primicerio [a]

[a] Dipartimento di Matematica "Ulisse Dini", Università degli Studi di Firenze, 55015 Firenze, Italy

[b] Departamento de Matemática Aplicada a los Recursos Naturales, ETSI de Montes, Universidad Politécnica de Madrid, 28040-Madrid, Spain

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ABSTRACT

We investigate optimal strategies to defend valuable goods against the attacks of a thief. Given the value of the goods and the probability of success for the thief, we look for the strategy that assures the largest benefit to each player irrespective of the strategy of his opponent. Two complementary approaches are used: agent-based modeling and game theory. It is shown that the compromise between the value of the goods and the probability of success defines the mixed Nash equilibrium of the game, that is compared with the results of the agent-based simulations and discussed in terms of the system parameters.

1. Introduction

In this note we shall study a particular example of how to protect a property from criminal actions. More precisely, we consider a classical security problem, which in its arguably simplest form can be formulated as follows: one security guard has to protect two different sites, where valuable goods are deposited, from the attack of a criminal. Each side, guard and criminal, has to select a strategy that best fits their own purpose: catching a pricey booty (and getting away with it) in one case, and keeping goods safe (and perhaps getting rid of marauders) on the other.

General qualitative results can be obtained by using a game theory approach [1–3]. Essentially, game theory seeks the best strategy a player has to play to optimize a certain payoff, for instance the largest benefit. Our problem can be defined as a zero-sum game with a payoff matrix that depends on the value of the sites and the probabilities of success for every choice of the players (guard and criminal). This is done in detail in Section 2. To shed more light on this problem, in Section 3, we address the simulation of a dynamic scenario where one guard and one thief move and fight each other for the booty. Unfortunately, even for this simple problem, agent-based models require to consider specific factors explicitly and, consequently, they need include a large number of parameters [4]. Both approaches complement each other since the information obtained from one of them allows the calibration of some of the parameters involved in the other [5]. For instance, as it will be seen, the mixed Nash equilibrium will provide information about the preferences of the agents, whereas the strategies of the agents give estimations of the probabilities of success for each of the players.

The article is organized as follows: in Section 2, we define a 2×2 zero-sum game and study its equilibria in terms of the ratio of the value of the two sites and the probability of success of the thief for each of the four possible pairs of player choices. In Section 3 an agent-based approach is used to implement a spatial game that mimics the theoretical game. Finally, in Section 4 we compare the two approaches and conclude with some general considerations.

* Corresponding author. Tel.: +34 913367107; fax: +34 913363230.
E-mail address: juancarlos.nuno@upm.es (J.C. Nuño).

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2. The game of the two goods

Let us consider a simple mathematical game with two players: one thief R and one guard C. Since the benefits of the thief are the losses of the guard, the game is defined as a zero-sum one. The goal of the guard is to minimize his losses, whereas the goal of the thief is to maximize his benefits. Each player can choose between two sites A and B. Each choice constitutes one of their strategies.

It is assumed that the payoff each player obtains for each of the four possible pairs of strategies depends on both the probability of success of the thief if he chooses to attack site i and the guard chooses to protect site j (Π_{ij}) and the value of each site (α_i). Besides, it can be supposed that the thief's success is only prevented by the encounter with the guard and thus, $\Pi_{AB} = \Pi_{BA} = 1$. If we rename $\Pi_{AA} = \Pi_A$ and $\Pi_{BB} = \Pi_B$, the payoff matrix for the thief is given by[1]:

		Guard	
		A	B
Thief	A	$\alpha_A \Pi_A$	α_A
	B	α_B	$\alpha_B \Pi_B$

A solution of the game corresponds to a pair of strategies adopted by each player. Since each player seeks the best payoff, if one player plays at random, the other one could find a strategy that maximizes his benefits and then, in a zero-sum game, that maximizes the losses of the opponent. It is well known that for a zero-sum game at least a Nash equilibrium exists and it coincides with the one produced by maximin and minimax strategies. Indeed, minimizing the opponent's maximum payoff (minimax rule) is identical to minimizing one's own maximum loss and to maximizing one's own minimum gain (maximin rule) [6].

It seems reasonable to assume that (i) the thief R always prefers the site without surveillance (i.e. that one not chosen by the guard C) and (ii) the guard C always prefers going to the same site chosen by the thief R. The first condition means that:

$$\alpha_B > \alpha_A \Pi_A \;\Rightarrow\; \rho = \frac{\alpha_A}{\alpha_B} < \frac{1}{\Pi_A}$$
$$\alpha_A > \alpha_B \Pi_B \;\Rightarrow\; \rho = \frac{\alpha_A}{\alpha_B} > \Pi_B.$$

Therefore, condition (i) implies that:

$$\Pi_B < \rho < \frac{1}{\Pi_A}.$$

Condition (ii) imposes the trivial inequalities:

$$\alpha_A \Pi_A < \alpha_A \;\Rightarrow\; \Pi_A < 1$$
$$\alpha_B \Pi_B < \alpha_B \;\Rightarrow\; \Pi_B < 1.$$

In order to compute the mixed Nash equilibrium, let us suppose that the thief R chooses a mixed strategy q_R, where q_R is the probability that R plays strategy A.[2] The payoff for the guard C is:

- $-\alpha_A \Pi_A q_R - \alpha_B (1 - q_R)$ if it chooses A and
- $-\alpha_A q_R - \alpha_B \Pi_B (1 - q_R)$ if it chooses B.

As it is well known, q_R represents a mixed Nash equilibrium if every C's strategy is a best response to it, that is, once R has chosen q_R, C gets the same payoff for every strategy he chooses. Therefore,

$$-\alpha_A \Pi_A q_R - \alpha_B (1 - q_R) = -\alpha_A q_R - \alpha_B \Pi_B (1 - q_R).$$

By solving this equation we obtain the mixed Nash equilibria as a function of ρ (Fig. 1):

$$q_R^* = \begin{cases} 0 & \text{if } \rho < \Pi_B \\ \left[0, \dfrac{1 - \Pi_B}{1 - \Pi_A \Pi_B}\right] & \text{if } \rho = \Pi_B \\ \dfrac{1 - \Pi_B}{(1 - \Pi_A)\rho + (1 - \Pi_B)} & \text{if } \Pi_B < \rho < \dfrac{1}{\Pi_A} \\ \left[\dfrac{\Pi_A (1 - \Pi_B)}{1 - \Pi_A \Pi_B}, 1\right] & \text{if } \rho = \dfrac{1}{\Pi_A} \\ 1 & \text{if } \rho > \dfrac{1}{\Pi_A} \end{cases}$$

1 Appendix A presents a brief analysis of the general case.

2 $q_R = 1$ corresponds to pure strategy A and $q_R = 0$ to pure strategy B (and similarly for C).