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Vehicular traffic flow through a series of signals with cycle time generated by a logistic map

Takashi Nagatani *, Naoki Sugiyama

Department of Mechanical Engineering, Division of Thermal Science, Shizuoka University, Hamamatsu 432-8561, Japan

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ABSTRACT

We study the dynamical behavior of vehicular traffic through a series of traffic signals. The vehicular traffic is controlled with the use of the cycle time generated by a logistic map. Each signal changes periodically with a cycle time, and the cycle time varies from signal to signal. The nonlinear dynamic model of the vehicular motion is presented by a nonlinear map including the logistic map. The vehicular traffic exhibits very complex behavior on varying both the cycle time and the logistic-map parameter a. For $a > 3$, the arrival time shows a linear dependence on the cycle time. Also, the dependence of vehicular motion on parameter a is clarified.

1. Introduction

Physics, other sciences and technologies meet at the frontier area of interdisciplinary research. The concepts and techniques of physics are being applied to such complex systems as transportation systems. Recently, transportation problems have attracted much attention in the fields of physics [1–5]. Traffic flow, pedestrian flow, and bus-route problems have been studied from a point of view of statistical mechanics and nonlinear dynamics [6–19]. The jams, chaos, and pattern formation are typical signatures of the complex behavior of transportation.

Mobility is nowadays one of the most significant ingredients of a modern society. In urban traffic, vehicles are controlled by traffic signals to give priority to one road where roads meet at crossings. In real traffic, the vehicular traffic flow depends highly on the control of traffic signals. Optimizing traffic lights for city traffic has been studied by using the CA traffic model and the optimal velocity model [20,21]. The effect of signal control strategy on vehicular traffic has been clarified. It has been shown that city traffic controlled by traffic signals can be reduced to a simpler problem of a single-lane highway with a few signals. There have been studies of vehicular traffic controlled by a few traffic signals. It has been concluded that the periodic traffic does not depend on the number of traffic lights.

Very recently, a few studies have been made on the traffic flow of vehicles moving through an infinite series of traffic lights with the same interval. The effect of cycle time on vehicular traffic has been clarified [22–28].

Generally, traffic lights are controlled by either synchronized or green-wave strategies. In the synchronized strategy, all the signals change simultaneously and periodically; the phase shift has the same value for all signals. In the green-wave strategy, the signal changes with a certain time delay between the signal phases of two successive intersections. The change of traffic lights propagates backward like a green wave. Thus, the vehicular traffic flow is controlled by varying the phase shift of the signals. More generally, the traffic signal can be controlled by means of the phase shift (offset time), cycle time, and split time.

* Corresponding author.
E-mail address: tmtnaga@ipc.shizuoka.ac.jp (T. Nagatani).

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The operator will be able to control the traffic signal by the use of another strategy. Specifically, one can manage the cycle times of signals. One will be able to control the vehicular traffic by changing the cycle time from signal to signal. Until now, vehicular traffic flow has been studied only in such a case that all signals have the same value for the cycle time. The study of vehicular traffic through a sequence of signals with the cycle time varying from signal to signal has been unknown.

In this paper, we study vehicular traffic flow through an infinite series of signals with cycle time varying from signal to signal. We apply a logistic map to the signal's strategy. We control the cycle time by using the logistic map. We present a nonlinear dynamic model for the vehicular motion through the series of traffic signals. We investigate the dynamical behavior of the vehicular traffic. We clarify the dependence of the vehicular motion through the sequence of signals on both the cycle time and the logistic-map parameter.

2. Nonlinear dynamic model

We consider the motion of vehicles going through an infinite series of traffic signals. We restrict ourselves to vehicular traffic at a low density. At a low density, vehicles move almost with a maximal speed. A vehicle is not affected by other vehicles. Therefore, we study the motion of a single vehicle. The traffic signals are numbered, from upstream to downstream, by $1, 2, 3, \ldots, n, n+1, \ldots$. The position of a signal is indicated by n. The traffic signals are positioned with the same interval on a roadway; the interval between signals $n-1$ and n is indicated by l. Signal n changes periodically with period $t_s(n)$. The cycle time varies from signal to signal. Period $t_s(n)$ is called the cycle time of signal n. The vehicle moves with mean speed v between one traffic light and the next.

In the synchronized strategy, all the traffic lights have the same value of cycle time and change simultaneously from red (green) to green (red) with a fixed time period $(1-s_p)t_s(s_pt_s)$. The period of green is s_pt_s and the period of red is $(1-s_p)t_s$. Fraction s_p represents the split which indicates the ratio of green time to cycle time.

The signal timing is controlled by the offset time t_{offset}. The offset time means the difference of phase shifts between two successive signals. In the green wave strategy, the phase shift of signal n is given by $t_{\text{phase}}(n) = nt_{\text{offset}}$, where the phase shift is indicated by $t_{\text{phase}}(n)$. Then, the signal switches from red to green in a green wave way. We restrict ourselves to the case that the phase shift of signals is zero. Here, we control the cycle time of signals with the use of a logistic map. The cycle time varies with signal n. When the logistic-map parameter a is higher than the critical value $a_c = 3.56$, the cycle time changes irregularly from signal to signal.

When a vehicle arrives at a traffic signal and the signal is red, the vehicle stops at the position of the traffic signal. Then, when the traffic signal changes from red to green, the vehicle goes ahead. On the other hand, when a vehicle arrives at a traffic signal and the signal is green, the vehicle does not stop and goes ahead without changing speed.

We define the arrival time of the vehicle at traffic light n as $t(n)$. The arrival time at traffic light $n+1$ is given by

$$t(n+1) = t(n) + l/v + (r(n) - t(n))\, H\left(t(n) - (\text{int}(t(n)/t_s(n))t_s(n)) - s_pt_s(n)\right)$$
$$\text{with } r(n) = (\text{int}\,(t(n)/t_s(n)) + 1)\cdot t_s(n), \tag{1}$$

where $H(t)$ is the Heaviside function: $H(t) = 1$ for $t \geq 0$ and $H(t) = 0$ for $t < 0$. $H(t) = 1$ if the traffic signal is red, while $H(t) = 0$ if the traffic signal is green. l/v is the time it takes for the vehicle to move between signals n and $n+1$. $r(n)$ is such time that the traffic signal just changed from red to green. The third term on the right-hand side of Eq. (1) represents such time that the vehicle stops if traffic signal n is red. The number n of iterations increases by one when the vehicle moves through the traffic signal. The iteration corresponds to proceeding on the highway.

By dividing the time by the characteristic time l/v_0, one obtains a nonlinear equation of dimensionless arrival time:

$$T(n+1) = T(n) + v/v_0 + (R(n) - T(n))\, H\left(T(n) - (\text{int}(T(n)/T_s(n))T_s(n)) - s_pT_s(n)\right)$$
$$\text{with } R(n) = (\text{int}\,(T(n)/T_s(n)) + 1)\cdot T_s(n), \tag{2}$$

where $T(n) = t(n)v_0/l$, $R(n) = r(n)v_0/l$, and $T_s(n) = t_s(n)v_0/l$.

Cycle time $T_s(n)$ is generated via the logistic map

$$T_s(n) = T_{s,0}f(n) \quad \text{with } f(n+1) = af(n)\,(1-f(n))\,, \tag{3}$$

where $f(n)$ is the logistic map and $T_{s,0}$ is the standard cycle time.

For $0 < a < 3$, the logistic map has a stable fixed point x_f. Then, all signals have the same value $T_{s,0}x_f$ of cycle time after a sufficiently large n. When a is higher than 3.0, periodic doubling occurs and the map displays chaos via a pitchfork bifurcation. The transition points at the periodic doubling are represented by $a_1, a_2, a_3 \ldots$. For $a_1 < a < a_2$, the cycle time $T_s(n)$ varies with period 2 after a sufficiently large n, and is either $T_{s,0}x_1$ or $T_{s,0}x_2$. Fig. 1(a) shows a schematic illustration of the signal's configuration for $a_1 < a < a_2$ after a sufficiently large n. Thus, the vehicle goes ahead through a series of two kinds of signal. For $a_2 < a < a_3$, the cycle time $T_s(n)$ varies with period 4 after a sufficiently large n, and takes values $T_{s,0}x_1$, $T_{s,0}x_2$, $T_{s,0}x_3$, and $T_{s,0}x_4$. Fig. 1(b) shows a schematic illustration of the signal's configuration for $a_2 < a < a_3$ after a sufficiently large n. The vehicle moves ahead through a series of four kinds of signal.

It will be expected that the vehicular motion can be controlled by the logistic map successfully. We study how the vehicular motion changes by varying both the standard cycle time $T_{s,0}$ and the logistic-map parameter a.