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A model of the effects of authority on consensus formation in adaptive networks: Impact on network topology and robustness

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ABSTRACT

Opinions of individuals in real social networks are arguably strongly influenced by external determinants, such as the opinions of those perceived to have the highest levels of authority. In order to model this, we have extended an existing model of consensus formation in an adaptive network by the introduction of a parameter representing each agent's level of 'authority', based on their opinion relative to the overall opinion distribution. We found that introducing this model, along with a randomly varying opinion convergence factor, significantly impacts the final state of converged opinions and the number of interactions required to reach that state. We also determined the relationship between initial and final network topologies for this model, and whether the final topology is robust to node removals. Our results indicate firstly that the process of consensus formation with a model of authority consistently transforms the network from an arbitrary initial topology to one with distinct measurements in mean shortest path, clustering coefficient, and degree distribution. Secondly, we found that subsequent to the consensus formation process, the mean shortest path and clustering coefficient are less affected by both random *and* targeted node disconnection. Speculation on the relevance of these results to real world applications is provided.

1. Introduction

Modelling interacting agents in social networks, and simulations of opinion formation processes in these models is currently an active field of research in statistical physics. For example, the 'bounded confidence model' of opinion formation with continuous opinions has been studied extensively [1–6]. However, the impact of an adaptively changing social network within that framework has only recently been introduced, and it was shown, for example, that convergence to a small number of opinion clusters is more likely when links within the network change adaptively [4,5]. While complex network topologies have been studied for the bounded confidence model [7], their significance in the context of adaptive networks is unknown.

A considerable amount of evidence has been generated which supports the predominance of complex networks amongst naturally evolved and human designed systems [8,9]. Such work includes research into network topology and biological evolution [10,11], work on 'small-world' networks [12] showing the importance of 'random links' for connectivity, and research supporting the theory that natural networks tend to evolve toward having heavy tailed degree distributions, e.g. 'scale-free networks' [13].

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Barabási and Albert have argued that the predominance of scale-free networks may be due in part to its natural robustness to network perturbations [13,8]. Such networks tend to be robust in terms of metrics such as degree distribution, clustering coefficient, and mean shortest path, when randomly chosen nodes are removed. However, it is also known that disconnection of high degree nodes (often referred to as hubs) can have catastrophic outcomes on these same metrics [14,15].

Many network simulation studies are based on the construction of constantly evolving networks, i.e. networks where nodes are added and removed on a continuing basis. The idea is that nodes are added to the network with a 'preferential attachment' bias, where nodes with a larger degree have a higher probability of attracting new nodes, or due to a 'fitness' quotient whereby some nodes are considered to be more *effective* at attracting connections from other nodes [16,17]. Consequent research into construction and behaviour of networks has tended to focus on such models, or networks with a static population predetermined at the point of creation [18–22].

However, this raises an interesting question, one that while being the subject of some conjecture [13,23,24] does not appear to have an easily found definitive answer – why do networks adopt and maintain the topologies that they do? Evolutionary principles of robustness may explain this to some extent – that is, networks that can best cope with perturbations are more likely to flourish. But, what about dynamic networks – i.e., where connections between nodes are made and broken over time – with relatively static population sizes, where definitions of fitness are complex, and depend on the network's purpose? Could it be that for naturally evolving networks, the function of the network contributes to its structural properties, so that robustness is a by-product of function, and that in turn, the most robust structures emerge?

If so, perhaps some benefit to network robustness in designed systems can be provided if a method can be found for replicating such behaviour. This is especially the case where the network in question is deemed likely to be vulnerable to targeted node interference.

Recent sociophysics simulations, aimed at investigating opinion formation and consensus in terms of the emergence of global phenomena based solely on local interactions, have illustrated the importance of adaptability in the connections between agents in social networks [25,4,5]. In such work, an evolving random graph model is used to represent agents who currently communicate, and simulations of interactions between individuals are carried out, in order to map the difference between social groups that are able to dynamically re-form, and those which are not. As described in Section 2, the opinion dynamics closely follow the bounded confidence model [1]. Nodes in the model networks represent agents and edges in the model networks represent the potential for communication between pairs of nodes. Each agent has a continuously valued opinion that can be altered after an interaction between pairs of neighbours, provided the two neighbours' opinions are within some tolerance. Investigations within this model have included varying population sizes, as well as levels of adaptability – i.e., the likelihood of an agent cutting a link to a neighbor and creating a new one – as well as the number of interactions required to achieve the network's final state [4,5]. Simulation results indicated that global consensus can only be formed in a static model when agents have relatively high tolerances for opinion difference, whereas when the ability for nodes to break old relationships and form new ones is introduced, consensus with relatively small tolerances becomes likely [4,5].

From a sociological point of view, we propose that this model can be extended in two ways that enable an improved match with real social interactions. The first extension is the inclusion of a more realistic randomly varying opinion-convergence factor. The second is the inclusion of an external determinant, namely the *perceived authority disparity* between the agents in the social network. In considering the first point, there is general acceptance [26,27] that when a consensus is reached between individuals (or indeed groups), as often as not that consensus is not the result of an absolute meeting of the minds. It is rather a working agreement, which originates from, as well as facilitating, a more subtle convergence of actual opinion. The second point is discussed below in Section 2.2.2. We have previously shown that these adaptations provide notable, and arguably more sociologically representative outcomes for opinion convergence modelling [28], as has subsequent work that also underlines the impact of both a continuous opinion convergence factor, and external determinants [6].

After including these two factors, we investigated their impact on the final distribution of opinions, and also the significance of initial network topology for the consensus formation process [28]. We found that the addition of these two factors resulted in a more compact opinion spread, with less interactions being required for the network to converge to its final state, i.e. where no more topological changes could be made (see Fig. 1). We also found that initial network topology had no significant effect on the final opinion state, but also noticed that after consensus formation, some statistics of the final networks had similar values regardless of initial topology. It is this latter finding that we investigate in a detailed way in this paper.

Precisely how to model the topology of local and global social networks, as well as other networks occurring both naturally and by design, has been a topic of research for some time. From Milgram, in the 1960s [29], through Strogatz and Watts' seminal work [12] and to this day, it has generally been agreed that both social networks, and most other naturally occurring networks, are not entirely unstructured by nature. Indeed, there is a growing body of research that suggests these networks have a complex structure that includes high clustering coefficient, low mean shortest path length, and in some instances a heavy tailed degree distribution [13,8,30–37].

The initial network topology in Refs. [4,5] was an Erdős–Rényi network model. This produces an artificially random network, in the sense that the number of neighbours for each node does not vary much, and there is no local clustering. In this paper we compare and contrast consensus formation in simulations of adaptive networks whose initial state is one of three canonical networks: the Erdős–Rényi *random* network, the Watts–Strogatz *small-world* network [12] or the Barabási–Albert