Physica A 390 (2011) 2087–2095

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

How to introduce temperature to the 1D Sznajd model

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ARTICLE INFO

Article history:
Received 3 March 2010
Received in revised form 11 November 2010
Available online 23 February 2011

Keywords:
Equilibrium
Temperature
Detailed balance
Opinion dynamics

ABSTRACT

We investigate the possibility of introducing temperature to the one dimensional Sznajd model and propose a natural extension of the original model by including other types of interactions. We characterise different kinds of equilibria into which the extended system can evolve. We determine the consequences of fulfilling the detailed balance condition and we prove that in some cases it is equivalent to microscopic reversibility. We found the equivalence of the model to the standard (inflow) model with interactions up to next nearest neighbors. It is shown that under some constraints there exists a Hamiltonian compatible with the dynamics and its form resembles that of the 1D ANNNI model. It appears however, that the standard approach of constructing temperature from the Hamiltonian fails. In this situation we propose a simple definition of the temperature-like quantity that measures the size of fluctuations in the system at equilibrium. The complete list of zero-temperature degenerated cases as well as the list of ground states of the derived Hamiltonian are provided.

1. Introduction

The Sznajd model was originally introduced in Ref. [1] in order to describe the mechanisms of opinion change in the society. The basic element of this approach is the "social validation", a social phenomenon relying on the fact that usually two people sharing common opinion have much bigger influence on other people in a group than separate individuals. The model itself and its modifications have found numerous applications in sociophysics (see Ref. [2] for a recent review), marketing [3,4], finance [5] and politics [6–8]. Apart from interest in social science, finance and politics, it also brings some new ideas to physics, as noted by Slanina and Lavicka [9]. Some theoretical aspects of the model were investigated in Refs. [9–11].

The Sznajd model provides a new scheme of interacting between the particles. The basic component of the Sznajd model is a lattice of Ising spins. Each spin is in one of two states: "up" or "down", like in the standard Ising model. Here the orientation of a given spin designates the opinion of an individual described by this spin, e.g. voting for or against in some political context. In the Sznajd model (and in other outflow models in general) we have the following dynamics. Within the common framework of random updating some chosen spins (their number depends on the variation of the model) influence their outer neighbors, as opposed to e.g. Glauber dynamics, where two spins affect the spin between them. In the first case we speak about *outflow dynamics*, as information flows outwards, and in the second case, we have *inflow dynamics*. Both kinds of models provide microscopic description of macroscopic changes in the system, e.g. phase transitions [12]. Some dynamical aspects of one- and two-dimensional versions of the Sznajd model were considered in Refs. [13–15]. Interesting scaling behavior of relaxation times for two-dimensional Sznajd model were reported in Ref. [16]. A simple underlying dynamical framework that drives global dynamics of the system was proposed for both Sznajd and Glauber zero-temperature models in Ref. [17].

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doi:10.1016/j.physa.2011.01.007

In almost all studies of outflow-type models the rules that govern dynamics are deterministic, and therefore these models are considered as zero-temperature. In the social context this means that all people behave in the same, predictable way, which is extremely rare in real situations. In general, different types of social response are possible [18], such as conformity, anticonformity, independence and congruence. These terms are related to various ways of changing mind in different conditions (e.g. conformity is the core of "social validation", as a group tries to make an individual following the group's opinion). Certainly human behavior is not predictable in 100% and with some probability one can expect any kind of social response. Therefore it would be reasonable and interesting to seek a model that incorporates some diversity or randomness in human activity. On the physical grounds this relates to a question: *How to incorporate noise, or temperature, to the original model in order to allow for randomness inherent in many real systems?* Such a development of the Sznajd model will allow us to formulate some general remarks about types of equilibrium that can be reached by the outflow models.

There has been some effort in dealing with the noise in sociophysical models in general. The need of introducing the noise to such models was discussed in fundamental paper on social impact [19]. According to a review by Castellano et al. [2] the notion of temperature is used with success in social impact theory, where so called "social temperature" governs the transition rates of changing the agent's opinion (eq. (18) of Ref. [2]). In many recent papers [20–27] the notion of the social temperature is incorporated into the models in various ways. In one type of approach the temperature-like parameter T enters the system through the dynamical rule of the Metropolis type: the (unfavourable) change of the ith single site (agent, trader, individual, actor, etc.) state is performed with the probability $\exp[-W(\sigma_i)/T]$, where the payoff function W measures the preferences and plays the role of the Hamiltonian. However often the payoff function cannot be considered as a true Hamiltonian. The detailed meaning of the function $W(\sigma_i)$ depends on the model — e.g. in Refs. [21,22] it is the difference in opinion between the i-th agent and its neighbors, in Ref. [23] it is a fixed parameter related to the strength of nearest-neighbor interactions, in Ref. [25] it is a total opinion gathered by the given trader. In another approach [26] the external modulation, which corresponds to a periodic fashion wave observed in real situations, is treated as the temperature analog. In some studies of social models [27] the noise plays an important role, as it triggers a stochastic resonance in the system. In the above mentioned papers the dynamics of the model is inflow (the vicinity influences the single individual), but in the outflow models there is no studies of this kind.

In the original Sznajd model there is no room for the temperature, and so an extension of the model is needed. Here (in Section two) we propose such an extended model. Our framework comes from taking into account all possible configurations of the small vicinity of each spin (or individual). It is so general, that it contains some previously examined models [13,28,29] as special cases. The proposed generalisation was recently used in Ref. [30] to show a possible mechanism of spontaneous reorientations of the whole system (society), where such behaviour was not possible in the original model.

In order to introduce the notion of temperature in the model one should examine the equilibrium of the system first (the standard temperature is well defined in equilibrium only). The discussion of that issue is provided in Section three. In Section four we show the equivalence between our model and the inflow model with interactions up to next nearest neighbors and under some constraints we derive the appropriate Hamiltonian. The discussion of the ground states of this Hamiltonian is also provided. In the fifth section we propose a natural candidate for the temperature in the extended model. Conclusions are drawn in the last (sixth) section. In the first appendix we present a complete list of special (degenerated) cases of the extended model, for which the dynamics always ceases after some finite time. In the second appendix we show the detailed arguments leading from the detailed balance conditions to the constraints (9) and (10).

It should be also noticed that the results presented in this paper are rigorous and belong to very few contributions that analyze the outflow dynamics analytically [9,31,32].

2. The extended model

In the original Sznajd model [1] of outflow dynamics one considers a system of L Ising spins $S_i = \pm 1$ on one dimensional lattice with periodic boundary conditions. In each update two nearest neighbors are selected at random. Let us assume that spins i and $i+1$ were selected. If they are parallel ($S_i = S_{i+1}$), their outer neighbors' spins follow the spins of the selected pair (that is $S_{i-1} = S_{i+2} = S_i$). In the case of antiparallel alignment ($S_i \neq S_{i+1}$) two options are considered: either the outer spins acquire antiparallel orientation with respect to their closer neighbor ($S_{i-1} = S_{i+1}$ and $S_{i+2} = S_i$) or nothing is changed. The latter option was considered to be more natural and was often used in the literature, but some other rules (for antiparallel alignment) were also used, see e.g. Refs. [13,28,29]. The dynamics defined in this way is undoubtedly zero-temperature, as no random noise is present in the system. All initial states approach one of the two final ferromagnetic states (all spins up or all spins down) in a finite time. The distribution of the relaxation times of this process was investigated in Ref. [15].

The original model's dynamic rules act only on configurations of the type $\ldots A[BB]A \ldots$ (from now on the selected pair of spins, that affect their outer neighbors, will be put in square brackets), where $A = \pm 1$ and $B = \mp 1$ stand for different spin orientations. In the cases of antiparallel alignment of the selected spins and of all four spins being parallel in a row nothing happens. The natural extension of the model accounts for allowing spin changes in these cases. Even though there are some other ways of introducing the noise to the Sznajd model (e.g. Ref. [33]), the one proposed here seems the most natural.

We consider a 1D lattice with periodic boundary conditions (as in the original model). There are L Ising spins on the lattice, and the dynamics of the model is given by special rules of changing spins in a single update step. At each step two consecutive spins are chosen at random, and they influence their direct neighbors according to the rules. Let us consider all possible configurations of 4 consecutive spins (two middle spins in brackets will control the outcome of the update step).