Physica A 392 (2013) 3891–3902

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Forecasting the underlying potential governing the time series of a dynamical system ☆

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HIGHLIGHTS

- The paper represents a new method of time series analysis, potential forecasting.
- Forecast PDF is obtained by extrapolation of Chebyshev approximation coefficients.
- Forecast series is derived from forecast PDF using rejection sampling and sorting.
- The method is tested on artificial data, temperature, and on Arctic sea ice data.
- The method completes a "tipping point toolbox" developed by Livina et al. in 2007–2013.

ARTICLE INFO

Article history:
Received 17 December 2012
Received in revised form 17 April 2013
Available online 2 May 2013

Keywords:
Potential forecasting
Potential analysis
Time series analysis

ABSTRACT

We introduce a technique of time series analysis, potential forecasting, which is based on dynamical propagation of the probability density of time series. We employ polynomial coefficients of the orthogonal approximation of the empirical probability distribution and extrapolate them in order to forecast the future probability distribution of data. The method is tested on artificial data, used for hindcasting observed climate data, and then applied to forecast Arctic sea-ice time series. The proposed methodology completes a framework for 'potential analysis' of tipping points which altogether serves anticipating, detecting and forecasting nonlinear changes including bifurcations using several independent techniques of time series analysis. Although being applied to climatological series in the present paper, the method is very general and can be used to forecast dynamics in time series of any origin.

1. Introduction

Many dynamical systems in general, and geophysical subsystems in particular, lack analytical deterministic descriptions with fully developed physical models, being represented mainly by recorded time series. At the same time such systems may be of great public interest and societal impact, such as the current climate change with rising temperature records around the globe.

In these circumstances, powerful research tools may be provided by statistical time series analysis [1–3], which have found entry into the analysis of climate data from a general viewpoint [4] and also from a nonlinear dynamical system

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http://dx.doi.org/10.1016/j.physa.2013.04.036

viewpoint [5]. In particular, system dynamics can be approximated by means of simple generalised stochastic models, where uncertain or unknown variables are represented by stochastic components [6,7]. Of specific regard to the present paper are time series analysis methods that deal with correlations and scaling of fluctuations [8–13].

In previous papers [14–16], we have developed several time series techniques for anticipating and detecting 'tipping points' in trajectories of dynamical systems, with applications in climatology. Modified 'degenerate fingerprinting' [14] was introduced for early warning of critical behaviour in time series to allow one to anticipate an upcoming bifurcation or transition (climate tipping points [17]). This was based on 'degenerate fingerprinting' [18], where the decay rate in the series is monitored using lag-1 autocorrelation in an autoregressive model (AR1). Modified degenerate fingerprinting employs Detrended Fluctuation Analysis for the same purposes. For noisy time series, we developed the method of potential analysis [15,16], which derives the number of system states under the assumption of quasi-stationarity of a data subset. This can distinguish a transition, which may be a forced drifting of the record without structural changes in the fluctuations, from a bifurcation which happens when the underlying system potential (the system states that a climate variable may sample) changes in structure, e.g. instead of two potential wells one or three wells appear. A bifurcation is characterised by structural change in the dynamical system, whereas transitional series preserve the same structure of fluctuations.

If both techniques give indication of dynamical change, this denotes a genuine bifurcation. If modified degenerate fingerprinting indicates a change but potential analysis does not, this means a transition rather than a bifurcation, with no changes in the underlying system potential.

In this paper, we develop the methodology further, so that we become able to not only anticipate and detect, but also to forecast the time series dynamics. The skill of such a forecast will depend on several factors, in particular, whether the upcoming change will be gradual or abrupt, at what rate it will be happening and how the scaling properties of the stochastic component may change with time.

Here we outline the methodology, test it on artificial data, in several hindcast case studies, and provide a forecast of the dynamics of Arctic sea-ice extent in the nearest future.

2. Methodology

2.1. Potential analysis as the basis of the method

We consider a simple stochastic model with a polynomial potential U as an approximation of the system dynamics,

$$\dot{x}(t) = -U'(x) + \sigma\eta, \tag{1}$$

where $\dot{x}$ is the time derivative of the system variable $x(t)$ (time series of an observed variable), η is Gaussian white noise of unit variance and σ is the noise level. In the case of a double-well potential, it can be approximated by a polynomial of 4th order:

$$U(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x.$$

According to the Fokker–Planck equation for the dynamic evolution of the probability density function $p(x, t)$,

$$\partial_t p(x, t) = \partial_x[U'(x)p(x, t)] + \frac{1}{2}\sigma^2\partial_x^2 p(x, t), \tag{2}$$

its stationary solution is given by Gardiner [19],

$$p(x) \sim \exp[-2U(x)/\sigma^2]. \tag{3}$$

The potential can be reconstructed from time series data of the system as

$$U(x) = -\frac{\sigma^2}{2}\log p_{\mathrm{d}}(x), \tag{4}$$

which means that the empirical probability density p_{d} has the number of modes corresponding to the number of wells of the potential.

This simple approximative approach allowed us to reconstruct the system potential of various climatic records (see Ref. [16]). It works with remarkable accuracy for data subsets of length as short as 400–500 data points, demonstrating above 90% rate of accurate detection, as was shown in an experiment with artificial data. For data subsets of length above 1000 points it correctly detects the structure of the potential with a rate of 98% [16]. Potential analysis was introduced in Ref. [15] which is an open-access paper published by Copernicus.org; this makes it easily accessible for the broad readership and more details on the methodology can be found there.

Here we develop the potential method beyond its detection capability, such that we are able to forecast the behaviour of a time series on the basis of its potential. To that effect, we introduce an extrapolation technique that would use the potential structure of the time series with linear extrapolation of the coefficients of the approximating polynomials. To reduce the biases introduced during various stages of the potential analysis (due to kernel distribution approximation, further logarithmic transformation, noise estimation, and finally polynomial fits), we use the empirical probability density