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Morphological similarity of road networks and cracks

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HIGHLIGHTS

- We study the possible similarity of road networks and cracks in their morphology.
- A desiccation model with double-layered cellular meshes is introduced.
- Simulation reveals the various patterns of real road networks and cracks.
- Simulation indicates the similar mechanism acting on their formation.

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ABSTRACT

An investigation was made regarding to what extent road network patterns are reproduced by a crack model from a viewpoint that they seem to resemble crack patterns in morphology. A desiccation model using double-layered cellular meshes was considered with the parameters representing the anisotropy of the material and the coarseness of grains, together with the introduction of singularities in points and in lines. The model can generally reproduce the real crack patterns and the road network patterns of cities with characteristic morphology by appropriately choosing the values of parameters, indicating that the similar mechanism acts on the formation of road networks and cracks of material although the relevant scales of space time differ from each other. Factors which make the road networks more complex and irregular in morphology were also investigated.

1. Introduction

How have the road networks through which we move everyday been formed? The formation mechanism of road networks such as assembling sets of roads, or the inevitable appearance of their morphological pattern has scarcely been considered even in a qualitative manner. In fundamental processes regarding the movement of mankind and the resultant formation of road, several factors have been considered as the outlook to a destination and the work required for getting to the destination [1–4]. When we take all these factors into account, however, it seems to be unable to obtain a general view or an appropriate model for the formation of the road network.

When we observe maps of the road network around big cities, we are seized with a feeling that the complicated web pattern of roads may be originated from some natural phenomenon in spite of the fact that it is a certain result of human activity. With regard to the road network within 200 km around Paris, for instance, an irregular pattern of network is formed as a superposition of trunk roads radially extended from the center of Paris on the outskirts roads which are almost perpendicular to the radial ones and concentric in many folds surrounding Paris. With approaching the center of Paris, such a road pattern becomes minute and increases its complexity. It reminds us of just the pattern of crack formed on a glass plate when a bullet is shot on it.

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Although the road networks in medieval cities, or the shape and arrangement of the blocks of building and the interval between them appear at first glance to stochastically distribute and irregularly extend, they seem to have some sort of morphological regularity on close looking, which is common to all cities. In medieval cities such as Beaune, Toulouse, Albi, Milano, Bologna, Modena, Ferrara, etc., for instance, their central parts are generally held by a church or a castle standing on a slightly elevated hill, and small-scale building blocks irregularly extend outward from them or plazas near them as if they are the nuclei of the cities. Also in many Mediterranean cities in the Iberian Peninsula such as Sevilla, Cordoba, Valencia, Evola, etc., the church, the castle, the religious house or the palace, along with the castle wall, stand as an absolute and immovable point (or line) so that a plural number of roads radially extend from them and the road network is formed as a superposed structure of them on the plural number of roads almost perpendicularly crossing with them. These all indicate the church or the castle as a point where forces or stresses are accumulated on it, that is, a singular point, and the structural pattern of roads seems as if it is a crack pattern originating from such a singular point. On the other hand, in the area of Jewish residence in those Iberian Peninsula cities, complicated winding together with dead ends are also seen in the road structure. Such a morphology strongly reminds us of the crack appearing on a clay plate during the process of desiccation [5].

Moreover in the network structure of medina in old towns in Islamic cities such as Fes, Marrakech, Casablanca, etc., such complexity and irregularity can be said to reach an extremity. In those medinas, superposed structures of road are realized which consist of the plural number of main roads connecting public buildings such as the casbah, the mosque, the palace, the madrasa and the souq, the secondary roads almost perpendicularly branched from the main road, and the tertiary roads, often with dead ends, also perpendicularly branched from the secondary ones. Such an orthogonal arrangement of multiple constituents can also be seen in a Mediterranean city as Venezia which is unrelated to Islam, suggesting the participation of a certain hierarchical process during the formation of the road network. When we say the network structure as a resulting phenomenon of a hierarchical process, we are reminded of the mechanism of crack formation on clay plates and ceramics [6].

In the Islamic society cited above, residences are constructed by the guideline called Hadith [7,8]. According to this, a private residence is formed at the end of an already formed residential block on the micro and isolated spirit which regulates the individual relation only to the neighboring residences such as the local avoidance of malodor and noise, or the use of space above the road just in front of the residence. Namely a new road develops depending only on the microscopic condition at the top end of the road. Such a mechanism reminds us of Griffith's theorem of crack [9]. Moreover it well resembles the situation of some type of cellular automaton where a certain region stochastically develops under the subjection of some local condition finally to result in a macroscopic structure.

Thus, the road network as a resultant of human activity and the crack patterns in nature have a positive resemblance to each other, suggesting a possible interpretation by the similar mechanism. When we consider *social stress* as a sum of psychological stress originating from individuals in the society, such social stress increases as the root becomes to be a detour to their destination, whereas it decreases as a short cut is developed in such a manner as to minimize the effort to get to the destination. The realization of roads convenient for all people, therefore, means the reduction of social stress, and this corresponds just to the release of mechanical stress on a plane by forming cracks. The edifices such as the church, the school or the market draw the public so that the social stress accumulates around those functions. This makes them the social singularities from or to which roads are formed in a least-effort manner to or from the outer city area, just in the radial direction to release the social stress.

In what follows we therefore investigate the road network with a kinetic model of crack and study a problem what variation of the factors which make the crack can lead to the structure of the road network. We also discuss the probable fluctuations added on the ideal crack to lead to the real morphology of the road network.

2. Model

We adopt the cellular mesh model [10–13] as a simulation model for the crack formation and propagation. Considered here is a situation such that the cracks are formed on a water-containing square plane with sizes $(Lx, Ly) \equiv (L_0, L_0)$, which shrinks by the desiccation of water molecules. We then equally divide the plane into $n \times n$ meshes with a side length $\Delta L = L_0/n$. Every node of the mesh is assumed to be connected by bonds to the eight neighboring nodes as shown in Fig. 1. The node is a representative point of the material within the area of $(\Delta L)^2$, and the bond corresponds to a sort of elastic spring with a spring constant k. The material considered is of double layers with a surface and a base and with a thickness ΔL, and connected by the same bond as on the surface between the surface and the base. As the water molecules intermingled in the surface layer evaporate with time, the stress is gradually accumulated in the material. Such a situation may be represented by gradually increasing the spring constants of the surface bonds. The force acting on the surface node (i, j) at a time t is given by the sum of the stretching forces originating from the eight Moor bonds on the surface $\Sigma \vec{F}_{ij}^t$ and the similar stretching forces $\Sigma \vec{H}_{ij}^t$ by the bond connecting to the base plane. At a time $(t + \Delta t)$ after the evaporation of water molecules, the node (i, j) moves to a new position $\vec{X}(i, j)^{t+\Delta t}$ where those forces balance, that is

$$\vec{X}(i,j)^{t+\Delta t} = \vec{X}(i,j)^t + \frac{1}{4}\left(\sum \vec{F}_{ij}^t + \sum \vec{H}_{ij}^t\right) \cdot (\Delta t)^2 .$$

Here the mass of the node is assumed to be unity and a constant force is assumed to act during Δt with an average strength. The node on the base plane $\vec{X}_0\,(i, j)$ is also assumed to have a limited slippery characteristic to substrate with a relaxation