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Dynamical robustness analysis of weighted complex networks

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HIGHLIGHTS

- Analyzing dynamical robustness of weighted complex networks.
- Comparing different roles of low-degree and high-degree nodes.
- Heterogeneous networks can be vulnerable even to the failure of low-degree nodes.
- Equal robustness of heterogeneous and homogeneous networks also exists.

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ABSTRACT

Robustness of *weighted* complex networks is analyzed from nonlinear dynamical point of view and with focus on different roles of high-degree and low-degree nodes. We find that the phenomenon for the low-degree nodes being the key nodes in the heterogeneous networks only appears in weakly weighted networks and for weak coupling. For all other parameters, the heterogeneous networks are always highly vulnerable to the failure of high-degree nodes; this point is the same as in the structural robustness analysis. We also find that with random inactivation, heterogeneous networks are always more robust than the corresponding homogeneous networks with the same average degree except for one special parameter. Thus our findings give an integrated picture for the dynamical robustness analysis on complex networks.

1. Introduction

Over the past several years, there has been increasing interest in the newly emerging network science with the aim to uncover complicated relations of highly intertwined elements in complex systems [1–8]. Clearly the network properties, such as the robustness for small response to large stimuli and the sensitivity for large response to small stimuli, are of much concern [9–15]. Basically, complex networks can be classified as heterogeneous networks, such as scale-free networks [16], and homogeneous networks, such as random graphs [17] and small-world networks [18], according to their different distributions of degrees. It has been commonly recognized that heterogeneous networks are extremely vulnerable to the failure of hubs (high-degree nodes), but robust to randomly removing a fraction of nodes than homogeneous networks. In contrast to this structural robustness focusing on the topological variation with removal of some key nodes, the dynamical (or functional) robustness in terms of nonlinear dynamics can be more complicated. Here the dynamical robustness of complex networks is defined as the ability of a network to maintain its dynamical activity when a fraction

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of the dynamical components are deteriorated or functionally depressed but not removed. For example, very recently Tanaka et al. have shown that low-degree nodes (not high-degree nodes) mostly play a crucial role in the dynamical robustness of heterogeneous networks [19], namely, though heterogeneous network is still robust to random inactivation than homogeneous network, it is vulnerable to targeted inactivation of low-degree nodes (not high-degree nodes). This is a highly counterintuitive result, highlighting the difference between structural and dynamical robustness in complex networks. It is noticeable that similar phenomenon has been reported in a simple case study, which showed that stimuli on low-degree nodes are easier to break the dynamical steady state of network than those on high-degree nodes [20]. Moreover, it has been found that the determining role is played by low-degree nodes, in contrast to the intuition that hub nodes are important for spreading dynamics in the study of optimal contact process in complex networks [21].

These pioneering works on dynamical robustness have focused mainly on unweighted networks by assuming that the strength of the connections is uniform. As we know, however, many realistic networks are indeed weighted; some examples include world-wide airport network [6], scientist collaboration network [6], neuronal network [22], and brain network [23], just to name only a few. The dynamical behaviors on networks should be affected not only by the topology, but also by the strength of the connections. For instance, it has been found that although both heterogeneity of degrees and heterogeneity of weights of complex networks are prone to suppress complete synchronization, synchronization is considerably enhanced and becomes independent of both heterogeneities when the distribution of weights is suitably combined with the distribution of degrees [24–26]. Therefore, it should be significant to further generalize the study of Tanaka and coworkers [19] and clarify the roles of high and low degree nodes on dynamical robustness in weighted complex networks.

In this paper, we will investigate the dynamical robustness of weighted complex networks consisting of N diffusively coupled nonlinear oscillators with the dynamics of isolated node simulated by the classical Landau–Stuart system [27], the same as in Ref. [19]. Both unweighted and weighted networks will be studied with one single system parameter β tunable for different degrees of weighted connection. The value $\beta = 0$ denotes the original unweighted network and all other $\beta \neq 0$ denotes weighted network with a scaled weight of coupling. Here in the dynamical robustness analysis, we are interested in how the network sustains (or, oppositely, changes) its dynamical activity if a fraction of the dynamical components are deteriorated, in particular, how easily the global oscillatory behavior transits to the steady state if the dynamics of some nodes are locally changed from oscillatory (active) to non-oscillatory (inactive). We find that the crucial role of low-degree nodes in the heterogeneous networks can only happen in weakly weighted networks and for weak coupling. For the usual case, where heterogeneous networks are more vulnerable to the failure of high-degree nodes, it actually appears in much wider parameter regions: such as in weakly weighted networks ($1 > \beta \geq 0$) and for large coupling, and in strongly weighted networks ($\beta \geq 1$) for any coupling. In addition, we find that the heterogeneous networks are always more robust than homogeneous networks for any random inactivation (attack), except for one special parameter point: $\beta = 1$, under which the heterogeneous and homogeneous networks show the same ability of robustness. All these detailed comparisons of dynamical robustness for either heterogeneous or homogeneous networks, and either high-degree or low-degree nodes attacks have been well proved to be supported by our mathematical analyses, and thus are of significance to provide a general framework for the robustness analysis in weighted complex networks.

This paper is organized as follows: In Section 2, the dynamic model and computational method are introduced. The dynamical robustness of unweighted and weighted heterogeneous networks is studied in Sections 3 and 4, respectively. The heterogeneous network and homogeneous network are compared in Section 5. Furthermore, we give theoretical analyses for these numerical results in Section 6. In Section 7, our conclusion and discussions are given.

2. Model

The model of N diffusively coupled Landau–Stuart oscillators [27] in a weighted network can be described as follows:

$$\dot{Z}_j = (\alpha_j + i\Omega - |Z_j|^2)Z_j + K\sum_{k=1}^{N} W_{jk}A_{jk}(Z_k - Z_j), \quad j = 1, \ldots, N \tag{1}$$

where Z_j is the complex state variable and α_j is the intrinsic parameter of jth oscillator, Ω denotes the natural frequency of single oscillator, and K is the overall coupling strength. $A = (A_{jk})$ and $W = (W_{jk})$ are the adjacency matrix and weight matrix of the network, respectively. $A_{jk} = A_{kj} = 1$ if the nodes j and k are connected by a link, and otherwise $A_{jk} = A_{kj} = 0$; self-connection is not allowed: $A_{jj} = 0$. Thus the adjacency matrix A is symmetric and undirected.

The same as in Ref. [24], we take

$$W_{jk} = 1/k_j^{\beta}, \tag{2}$$

where $k_j = \sum_{k=1}^{N} A_{jk}$ is the degree of the jth node, and β is a tunable parameter: $\beta \neq 0$ indicates the network is weighted while it is unweighted when $\beta = 0$ [24]. With this expression, the weighted coupling is determined by the node degree and thus the network becomes asymmetric and directed. It has been reported that the synchronizability can be easily enhanced with the effect of weighted coupling for any given fixed network topology, and especially the synchronizability becomes maximal at exactly $\beta = 1$ [24]. Here following the same idea, we will analyze the network's dynamical robustness.